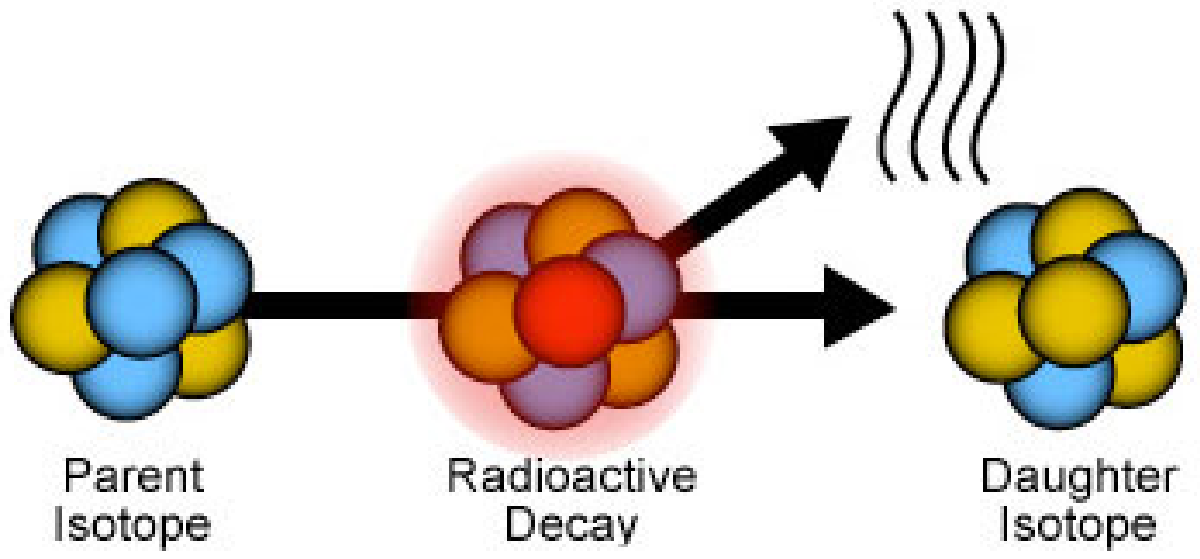
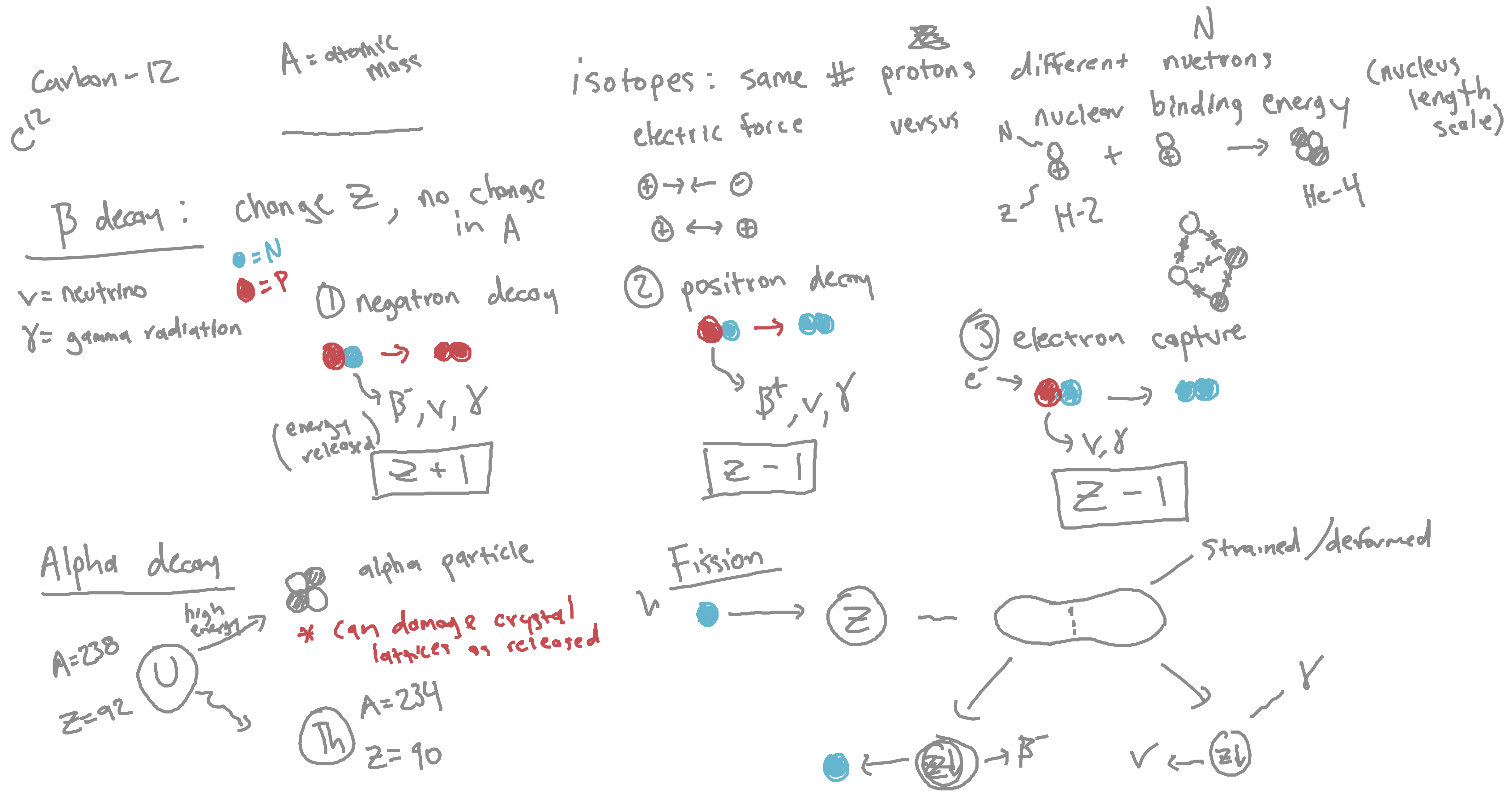


Lecture 11-13: Radioactive Decay

1. Mechanisms
2. The decay equation
    A. Isochrons
    B. Sm-Nd system
    C. Pb-Pb dating (age of the Earth)

We acknowledge and respect the lək̓ʷəŋən peoples on whose traditional territory the university stands and the Songhees, Esquimalt and W̱SÁNEĆ peoples whose historical relationships with the land continue to this day.

Mechanisms of radioactive decay.



The decay equation.

Rutherford + Soddy 1902

N = number of moles of an isotope

$\frac{dN}{dt} \propto N$

$\frac{dN}{dt} = -\lambda N$

proportionality constant

decay constant

$\ln x \Big|_{N_0}^{N}$

$\int \frac{dN}{N} = \int -\lambda \, dt$

$\ln N - \ln C = -\lambda t$

N_0 = initial concentration of N

$\ln \frac{N}{C} = -\lambda t$

$e^{-\lambda t} = \frac{N}{C}$

$Ce^{-\lambda t} = N$

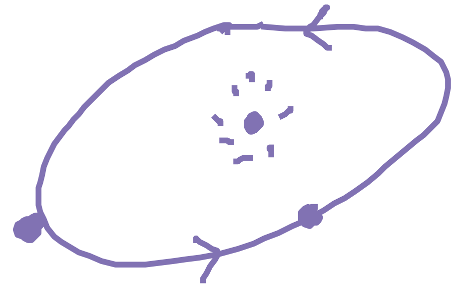
$N_0 e^{-\lambda t} = N$

Experiments to test "constant"

↳ high vs low T

↳ high vs low P

↳ high vs low Magnetic field



no change from gravity

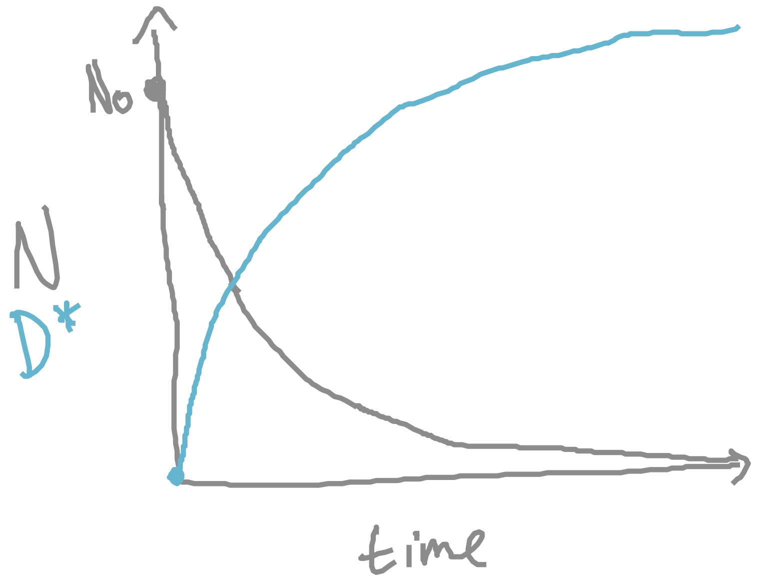
The decay equation.

measurable

$$N_0 e^{-\lambda t} = N$$

not measurable

measurable



N= parent isotope

D= descendant isotope

$$\left.\begin{aligned} N_0 e^{-\lambda t} &= N \\ N e^{\lambda t} &= N_0 \end{aligned}\right\} \text{reversible}$$

$D^* = N_0 - N$ Descendant created by the decay of N

$$D = \underset{\substack{\uparrow \\ \text{initial } D}}{D_0} + \underset{\substack{\uparrow \\ \text{generated } D}}{D^*}$$

$D^* = N e^{\lambda t} - N_0 e^{-\lambda t}$ * hard to measure absolute values, so ratio w/ stable isotope common

$$D^* = \underset{\substack{\uparrow \\ \text{measured}}}{N} (e^{\lambda t} - 1)$$

$$\frac{D}{X} = \frac{D_0}{X} + \frac{N}{X}(e^{\lambda t} - 1)$$

Half life

— time it takes half of N to decay

$$N = N_0 e^{-\lambda t}$$

$$\frac{N_0}{2} = N_0 e^{-\lambda t}$$

$$\frac{1}{2} = e^{-\lambda t}$$

$$\ln\frac{1}{2} = -\lambda t$$

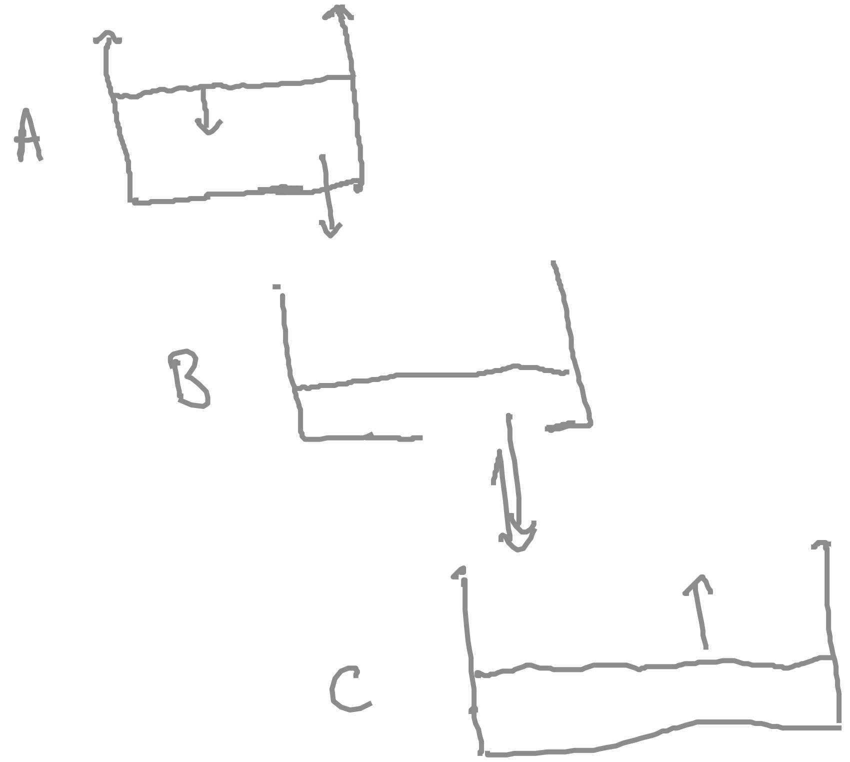
$$-\frac{\ln\frac{1}{2}}{\lambda} = t_{1/2} \text{ (half-life)}$$

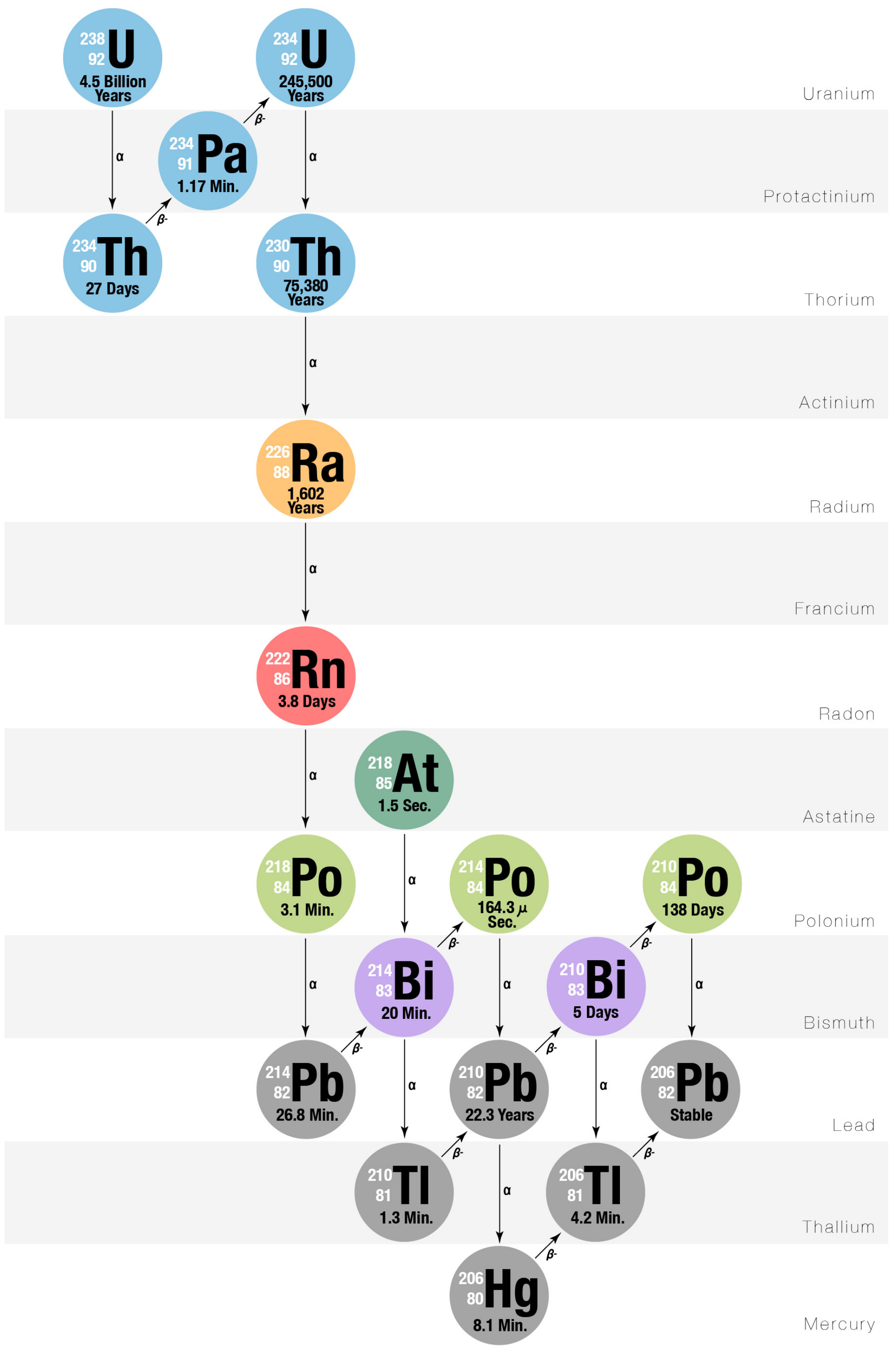
Decay chains:

$A \rightarrow B \rightarrow C$

Unstable

stable





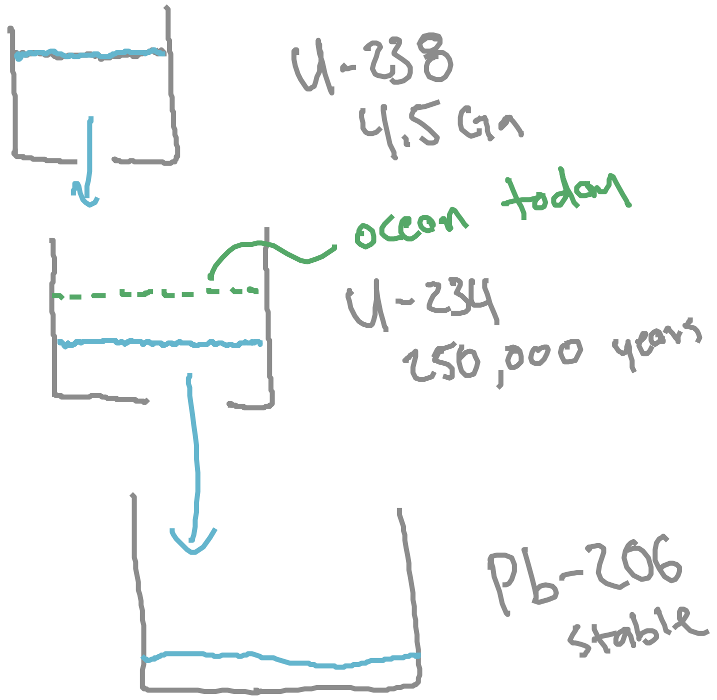
238 92 U 4.5 Billion Years
234 92 U 245,500 Years
234 91 Pa 1.17 Min.
234 90 Th 27 Days
230 90 Th 75,380 Years
226 88 Ra 1,602 Years
222 86 Rn 3.8 Days
218 85 At 1.5 Sec.
218 84 Po 3.1 Min.
214 84 Po 164.3 μ Sec.
210 84 Po 138 Days
214 83 Bi 20 Min.
210 83 Bi 5 Days
214 82 Pb 26.8 Min.
210 82 Pb 22.3 Years
206 82 Pb Stable
210 81 Tl 1.3 Min.
206 81 Tl 4.2 Min.
206 80 Hg 8.1 Min.
α
β-
Uranium
Protactinium
Thorium
Actinium
Radium
Francium
Radon
Astatine
Polonium
Bismuth
Lead
Thallium
Mercury

U-Series dating of corals

U-238
half-life 4.5 Ga
α - alpha
Th-234

— U-234

Slowest individual decay
is most important



stable descendant
isotope Pb-206

Isochrons

how much Descendant isotope does our sample start with?

↳ how do determine the age of a sample if we don't Know D_0?

Partition coefficients for N

$D_A^N = 4$

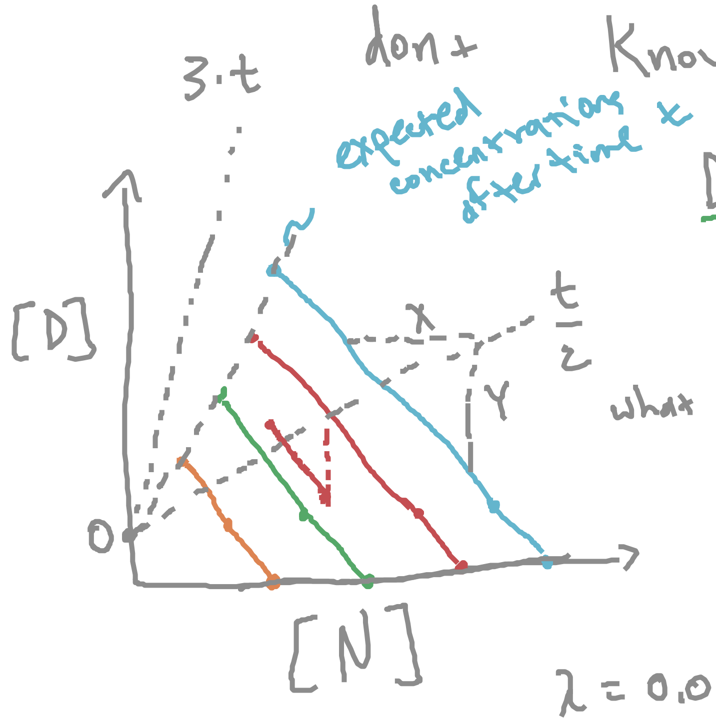
$D_B^N = 3$

$D_C^N = 2$

$D_D^N = 1$

$D_D^D = 0$

$N \rightarrow D$



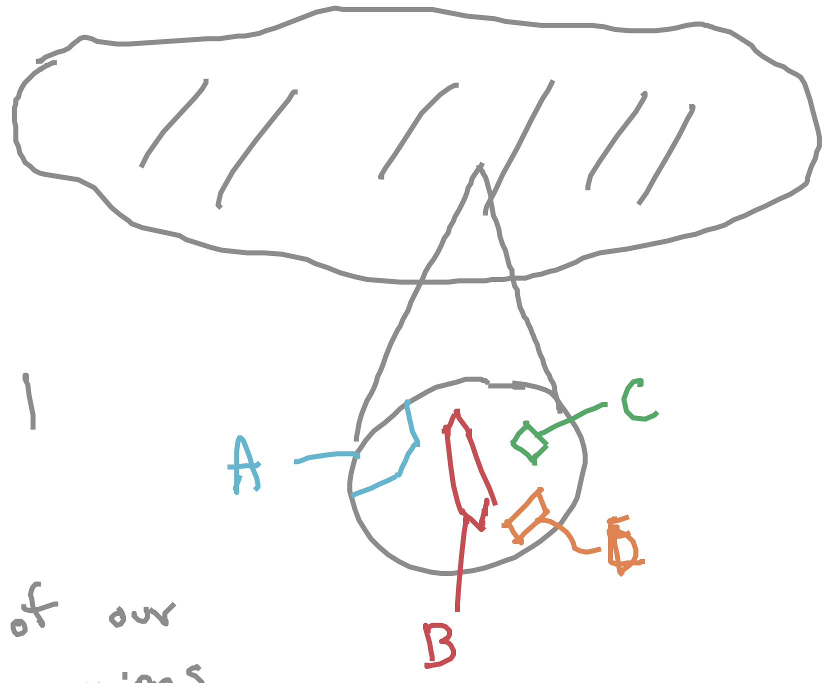
$$D = D_0 + N(e^{\lambda t} - 1)$$

$$Y = B + X\,M$$

↑ 0

what is $\frac{X}{Y}$? slope of 1

$\lambda = 0.01$



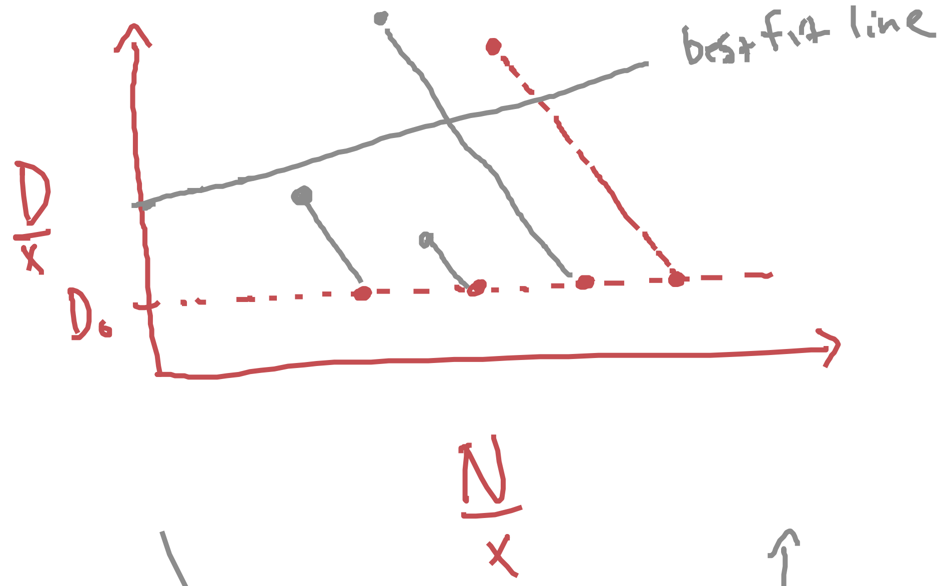
isochrons:
the slope of our sample observations is a function of time

if $m = 10$

$$10 = e^{\lambda t} - 1$$
$$11 = e^{\lambda t}$$
$$\ln 11 = \lambda t$$

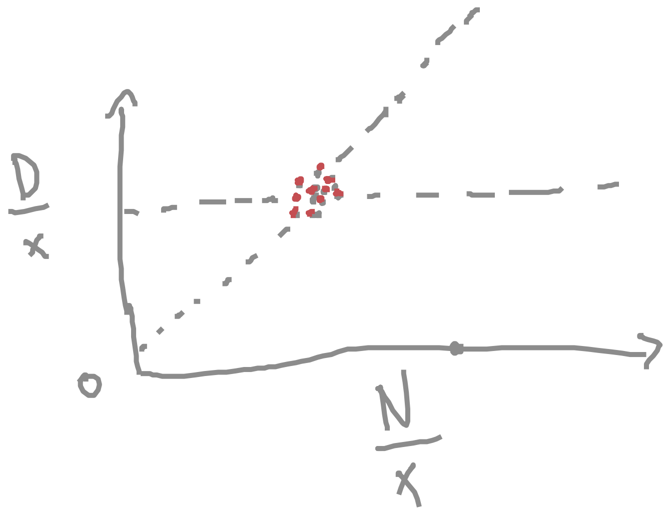
$$\ln e^x = x$$

if a suite of samples forms a line in $\frac{D}{N}$, then the samples may have the same age and initial D_0.



to use an isochron:

1. Samples have same age
2. Samples have the same initial Descendant isotope
3. D and N isotopes do not enter or leave the mineral or rock since formation (closed system)



it helps when the samples have very different N concentrations

Recall:

$$N_0 e^{-\lambda t} = N$$

Recall:

$$N_0 e^{-\lambda t} = N$$

In [3]:

```
N_0 = 1 # parent isotope initial
LAMBDA = 1e-4 # decay constant
t = np.linspace(0, 1e5, 1000) #time (from now to 100 thousand years ago)
N = N_0 * np.exp(-LAMBDA * t) # parent isotope
D = N_0 - N #descendant isotope
```

Recall:

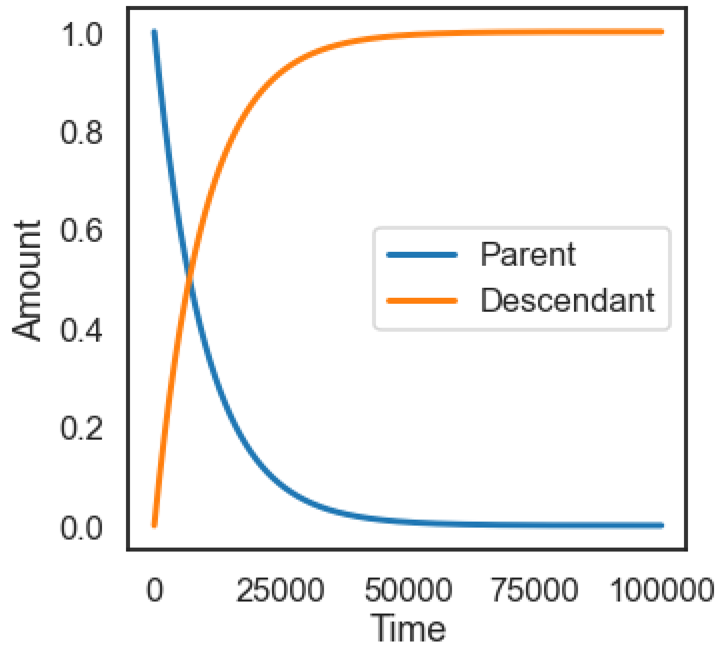
$$N_0 e^{-\lambda t} = N$$

In [3]:

```
N_0 = 1 # parent isotope initial
LAMBDA = 1e-4 # decay constant
t = np.linspace(0, 1e5, 1000) #time (from now to 100 thousand years ago)
N = N_0 * np.exp(-LAMBDA * t) # parent isotope
D = N_0 - N #descendant isotope
```

In [4]:

```
plt.figure(figsize=(5, 5))
plt.plot(t, N, label="Parent", alpha=1, lw=3)
plt.plot(t, D, label="Descendant", alpha=1, lw=3)
plt.legend(loc="best")
_ = plt.gca().set_xlabel("Time")
_ = plt.gca().set_ylabel("Amount")
```



Batch melting:

$$\frac{C_S}{C_0} = \frac{D}{F + D(1 - F)}$$

Batch melting:

$$\frac{C_S}{C_0} = \frac{D}{F + D(1 - F)}$$

In [10]:
```
def batch_S(F, D, Co): #batch melting equation for solid
    return (D*Co) / (F + D * (1 - F))

D_n = np.array([1,2,3,4]) #a list of partition coefficients
F = 0.8 #melt fraction
N_0 = batch_S(F, D_n, 1) #C_l for each F
```

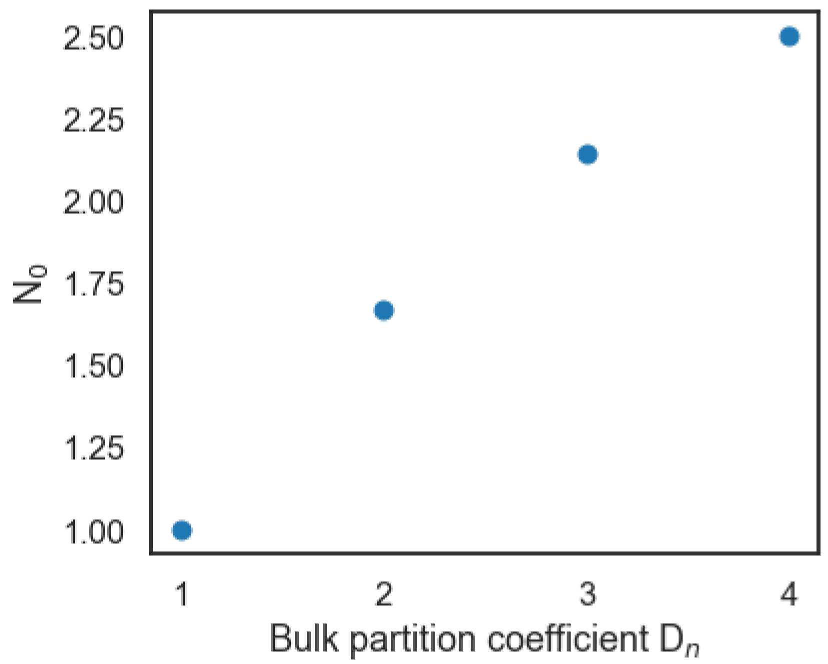
Batch melting:

$$\frac{C_S}{C_0} = \frac{D}{F + D(1 - F)}$$

In [10]:
```
def batch_S(F, D, Co): #batch melting equation for solid
    return (D*Co) / (F + D * (1 - F))

D_n = np.array([1,2,3,4]) #a list of partition coefficients
F = 0.8 #melt fraction
N_0 = batch_S(F, D_n, 1) #C_l for each F
```

In [11]:
```
plt.figure(figsize=(6, 5))
plt.plot(D_n, N_0, "o")
_ = plt.gca().set_xlabel("Bulk partition coefficient D$_n$")
_ = plt.gca().set_ylabel("N$_0$")
```



In [15]:

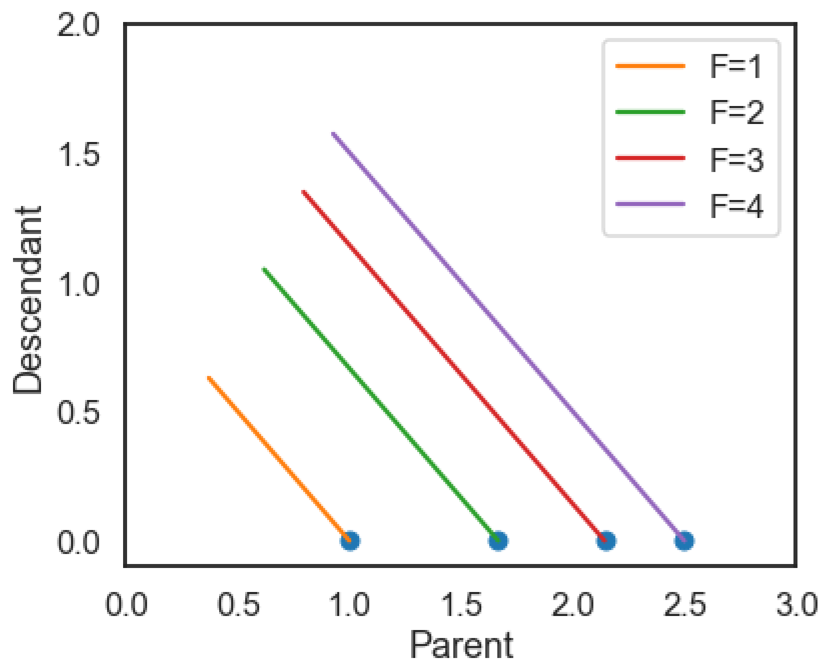
```
N = [n * np.exp(-LAMBDA * t[:100]) for n in N_0] # the decay equation for each starting N
C = [(n - NS) for n, NS in zip(N_0, N)] # the descendant isotope (initial minus current)
```

In [15]:

```
N = [n * np.exp(-LAMBDA * t[:100]) for n in N_0] # the decay equation for each starting N
C = [(n - NS) for n, NS in zip(N_0, N)] # the descendant isotope (initial minus current)
```

In [17]:

```
plt.figure(figsize=(6, 5))
plt.plot(N_0, [0, 0, 0, 0], "o", alpha=1)
_ = [plt.plot(a, b, label="F=" + str(c), alpha=1) for a, b, c in zip(N, C, D_n)]
plt.legend(loc="best")
_ = plt.gca().set_xlabel("Parent")
_ = plt.gca().set_ylabel("Descendant")
_ = plt.gca().set_xlim([0, 3])
_ = plt.gca().set_ylim([-0.1, 2])
```



In [18]:

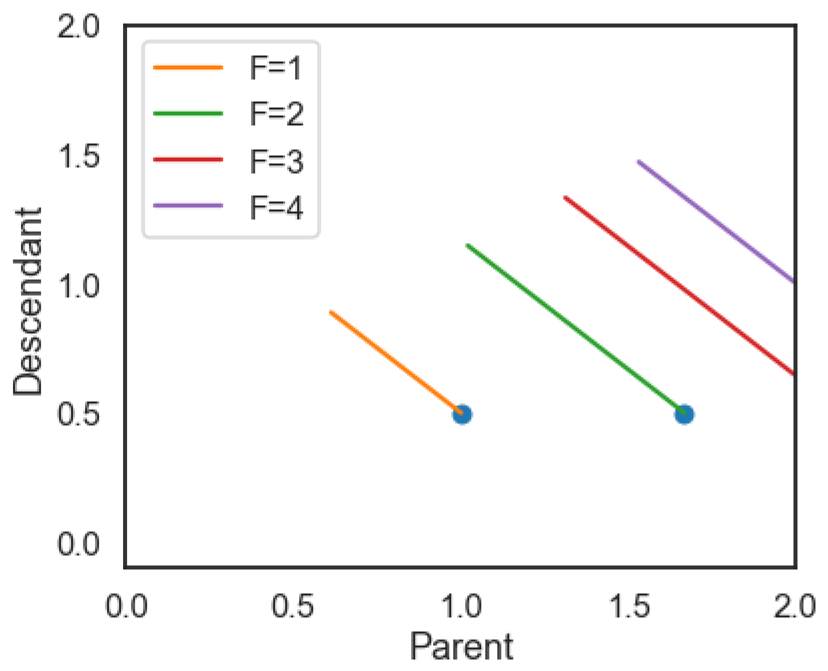
```
np.random.seed(12)
N = [n * np.exp(-LAMBDA * t[:50]) for n in N_0] # the decay equation for each starting N
Ic = 0.5*np.ones(4) # some fixed amount of initial child isotope
C = [i + (n - NS) for i, n, NS in zip(Ic, N_0, N)] # the descendant isotope (initial parent minus current)
```

In [18]:

```
np.random.seed(12)
N = [n * np.exp(-LAMBDA * t[:50]) for n in N_0] # the decay equation for each starting N
Ic = 0.5*np.ones(4) # some fixed amount of initial child isotope
C = [i + (n - NS) for i, n, NS in zip(Ic, N_0, N)] # the descendant isotope (initial parent minus current)
```

In [19]:

```
plt.figure(figsize=(6, 5))
plt.plot(N_0, Ic, "o")
_ = [plt.plot(a, b, label="F=" + str(c)) for a, b, c in zip(N, C, D_n)]
plt.legend(loc="best")
_ = plt.gca().set_xlabel("Parent")
_ = plt.gca().set_ylabel("Descendant")
_ = plt.gca().set_xlim([0, 2])
_ = plt.gca().set_ylim([-0.1, 2])
```



In [20]:

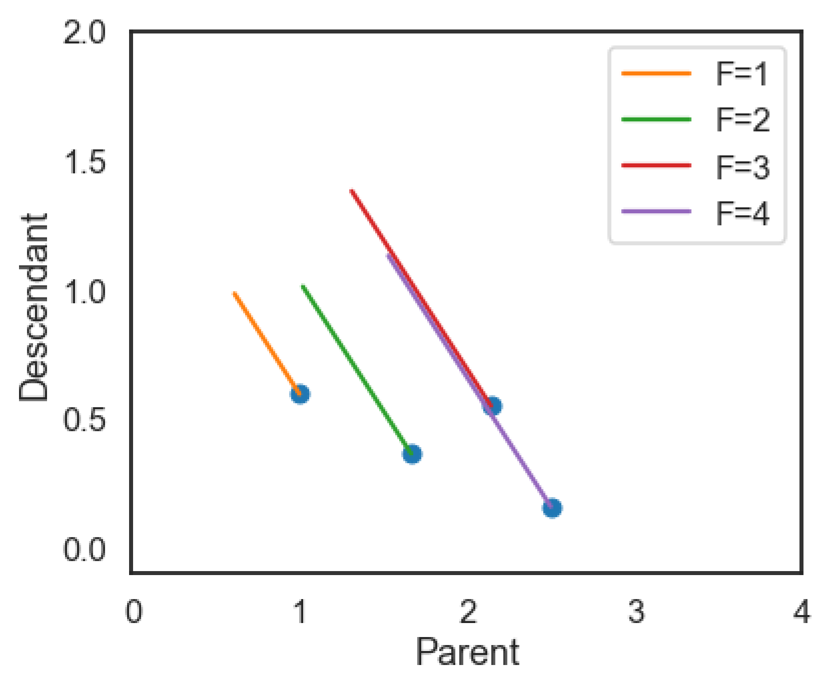
```
Ic = 0.5 + np.array(np.random.normal(0.0, 0.2, 4)) # some random amount of initial child isotope
C = [i + (n - NS) for i, n, NS in zip(Ic, N_0, N)] # the descendant isotope (initial parent minus current)
```

In [20]:

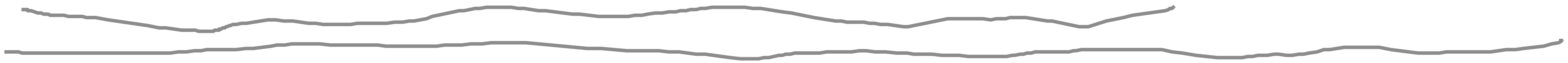
```
Ic = 0.5 + np.array(np.random.normal(0.0, 0.2, 4)) # some random amount of initial child isotope
C = [i + (n - NS) for i, n, NS in zip(Ic, N_0, N)] # the descendant isotope (initial parent minus current)
```

In [22]:

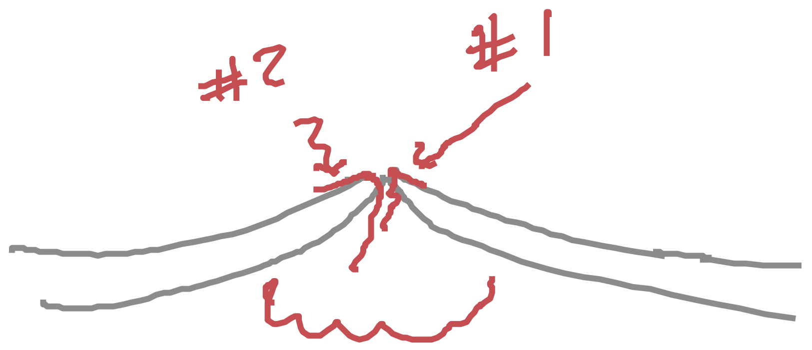
```
plt.figure(figsize=(6, 5))
plt.plot(N_0, Ic, "o")
_ = [plt.plot(a, b, label="F=" + str(c)) for a, b, c in zip(N, C, D_n)]
plt.legend(loc="best")
_ = plt.gca().set_xlabel("Parent")
_ = plt.gca().set_ylabel("Descendant")
_ = plt.gca().set_xlim([0, 4])
_ = plt.gca().set_ylim([-0.1, 2])
```



Utility



Review: any of our melting models <u>could</u> produce variations in P isotopes



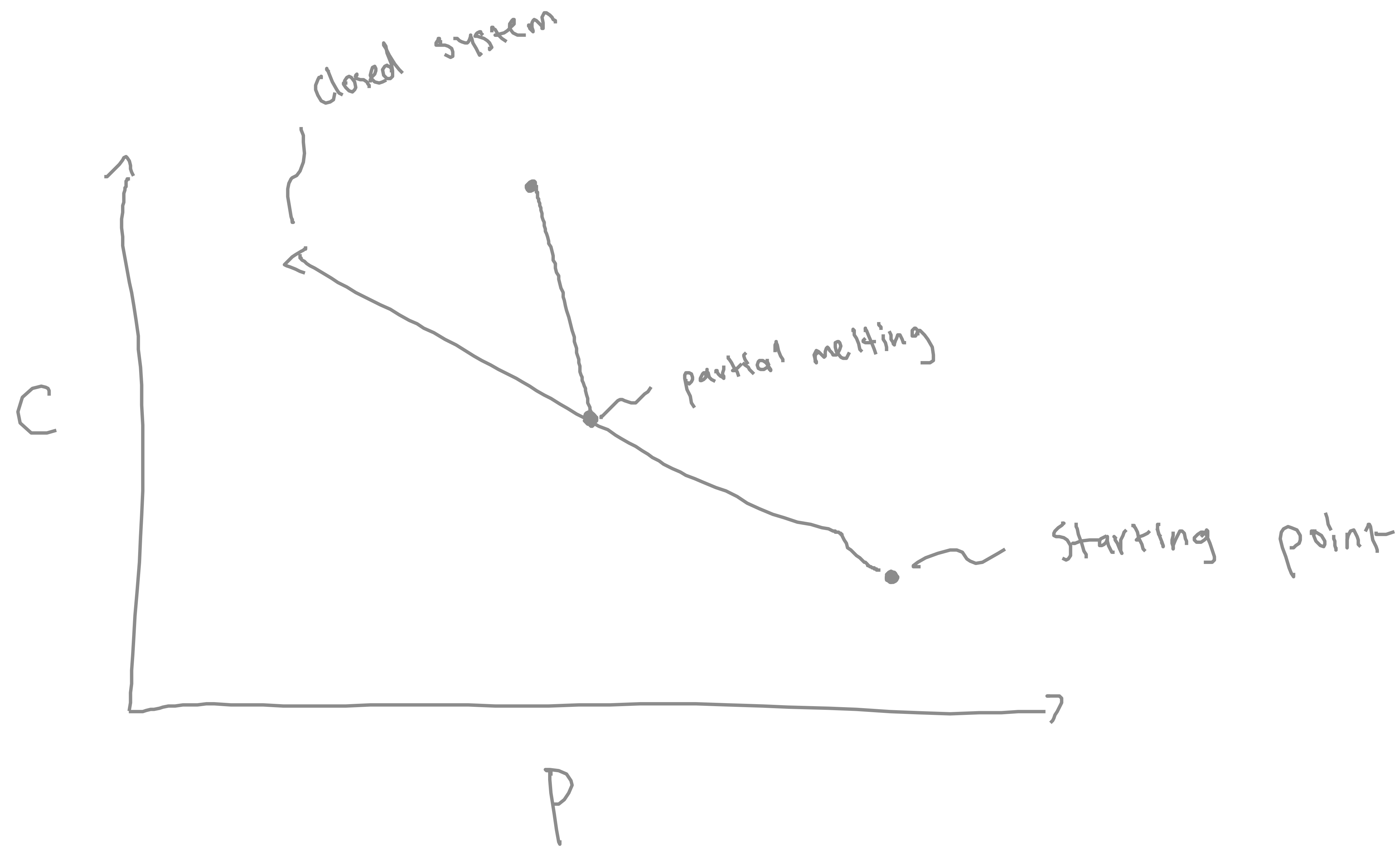
isochrons: $y = mx + b$ ← the decay equation reduces to this form with the following assumptions:

$m = e^{\lambda t} - 1$

- Same initial Daughter isotope concentration
- closed system
- Same age of samples

Sm-Nd decay and model ages (the chondritic uniform reservoir or CHUR)

^{147}Sm decays to ^{143}Nd through alpha decay with a decay constant of $\lambda^{147}=6.54 \times 10^{-12}$



Age of the Earth

~ 4.6 Ga

oldest ice ~ several million years

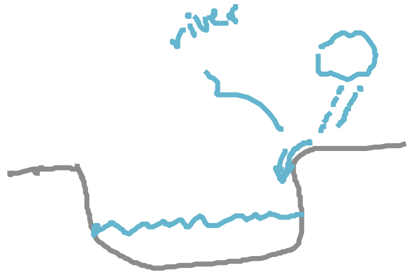
oldest mineral jack hills Australia ~ 4.0 Ga

↳ zircon

oldest rock ~ 3.8 Ga

historical attempts

#1

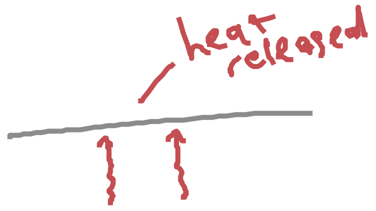


how long does it take to make the ocean salinity match observations?

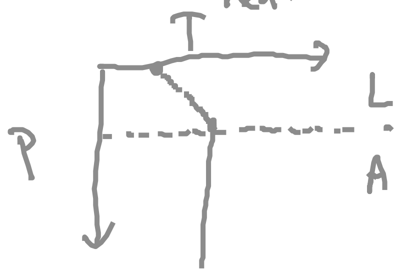
↳ way too young

↳ ions become rocks over time

#2



- assumed Earth ~~transports~~ heat through conduction
- predicted how long it takes Earth to cool to current heat release measurements



Age of the Earth

Pb/Pb dating

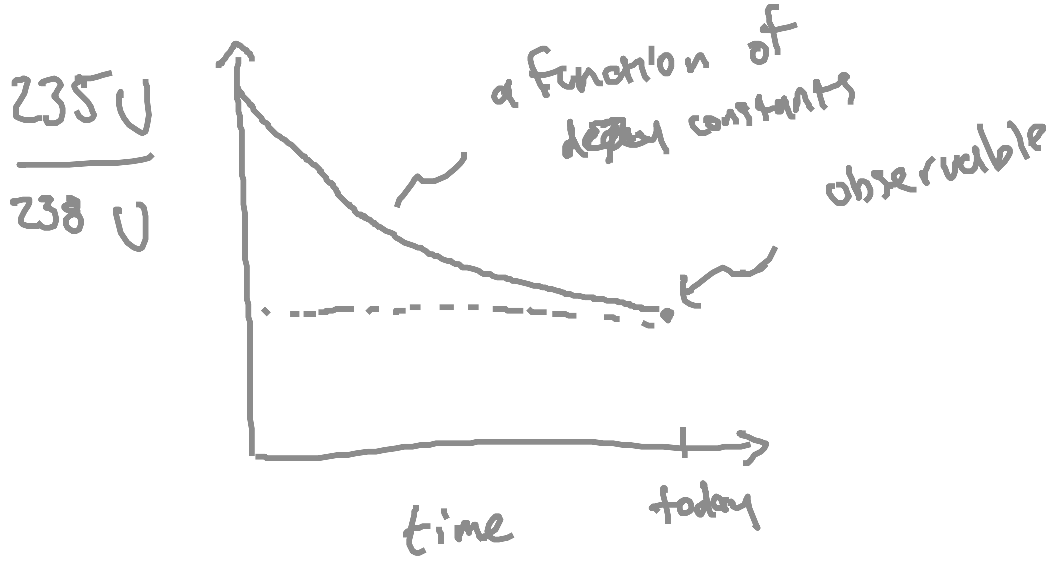
① $^{238}U \longrightarrow {}^{206}Pb$ $t_{1/2}$: 4.5 Ga

② $^{235}U \longrightarrow {}^{207}Pb$ $t_{1/2}$: 0.7 Ga

^{204}Pb = "common lead"

* nothing decays to ^{204}Pb

U is RLE



① $$\frac{^{206}Pb}{^{204}Pb} = \left(\frac{^{206}Pb}{^{204}Pb}\right)_0 + \frac{^{238}U}{^{204}Pb}\left(e^{\lambda_{238} t} - 1\right)$$

initial

② $$\frac{^{207}Pb}{^{204}Pb} = \left(\frac{^{207}Pb}{^{204}Pb}\right)_0 + \frac{^{235}U}{^{204}Pb}\left(e^{\lambda_{235} t} - 1\right)$$

subtract

Age of the Earth

ratio of $\frac{②}{①}$

ratio of ① and ②

$$\frac{\frac{^{207}Pb}{^{204}Pb} - \left(\frac{^{207}Pb}{^{204}Pb}\right)_0}{\frac{^{206}Pb}{^{204}Pb} - \left(\frac{^{206}Pb}{^{204}Pb}\right)_0} = \frac{^{235}U}{^{238}U} \cdot \frac{(e^{\lambda_{235} t} - 1)}{(e^{\lambda_{238} \cdot t} - 1)}$$

rearrange into

$Y = mX + mC_1 + C_2$

$Y = \frac{^{207}Pb}{^{204}Pb}$

$X = \frac{^{206}Pb}{^{204}Pb}$

$m = \frac{^{235}U}{^{238}U} \frac{(e^{\lambda_{235} t} - 1)}{(e^{\lambda_{238} t} - 1)}$

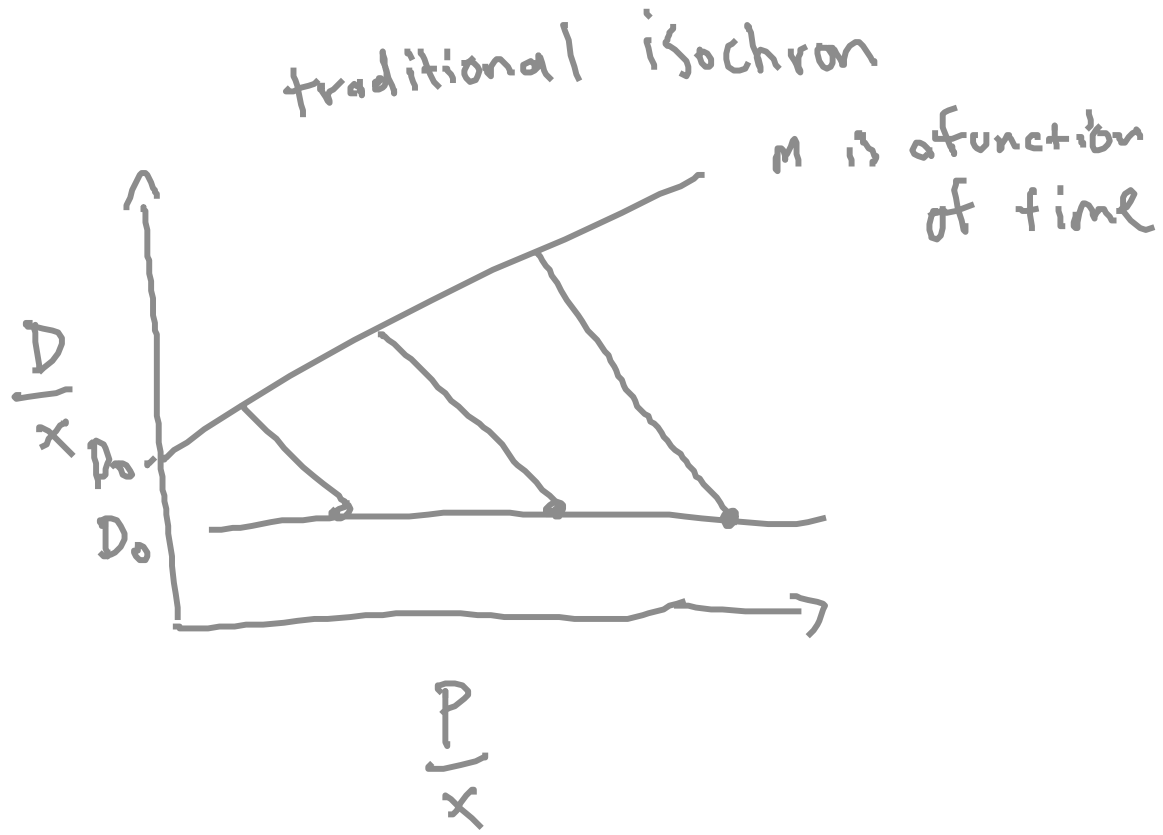
.... some algebra

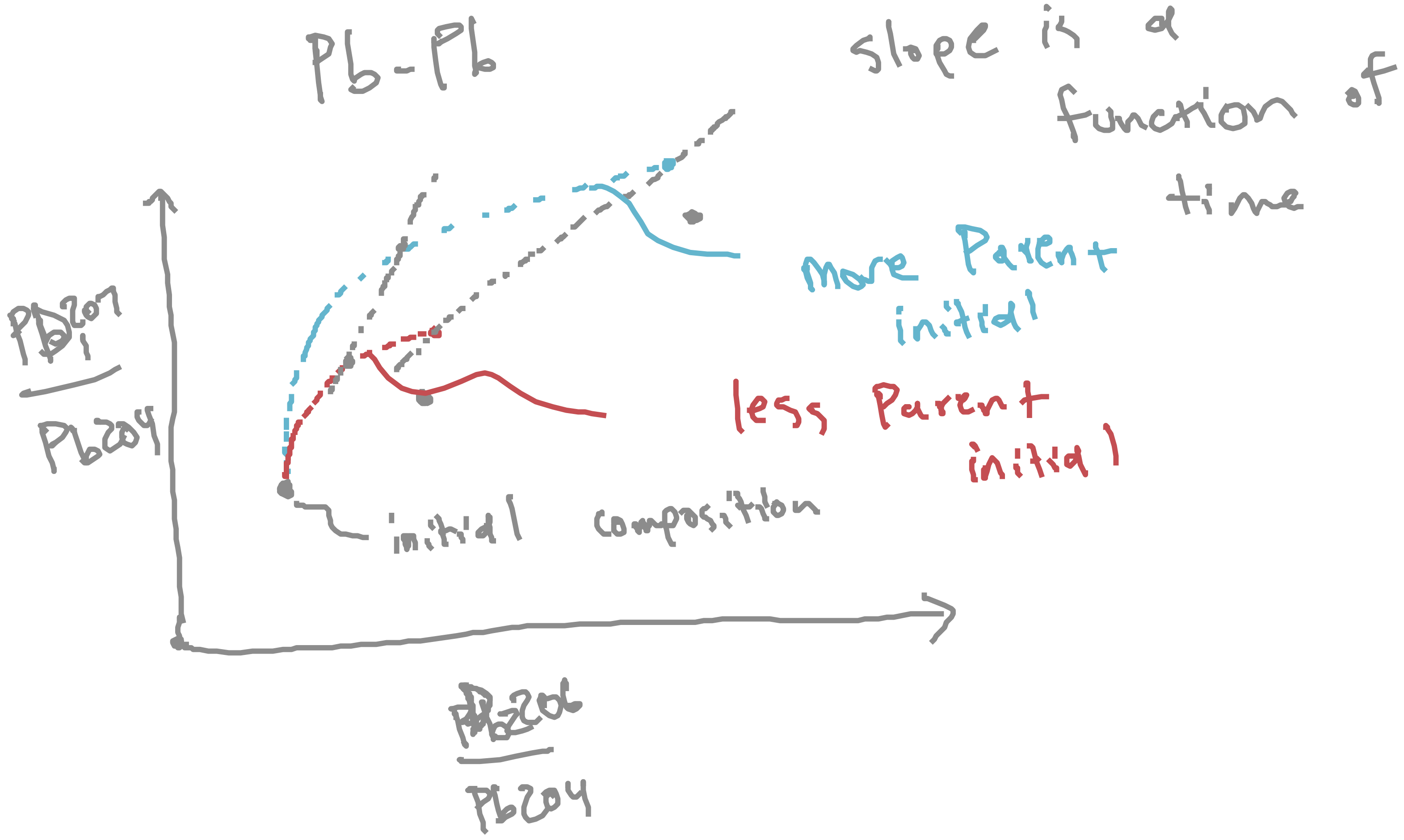
$$\frac{^{207}Pb}{^{204}Pb} = \frac{^{235}U}{^{238}U} \frac{(e^{\lambda_{235} t} - 1)}{(e^{\lambda_{238} t} - 1)} \cdot \frac{^{206}Pb}{^{204}Pb} + \frac{^{235}U}{^{238}U} \frac{(e^{\lambda_{235} t} - 1)}{(e^{\lambda_{238} t} - 1)} \cdot \left(\frac{^{206}Pb}{^{204}Pb}\right)_0 + \left(\frac{^{207}Pb}{^{204}Pb}\right)_0$$

$Y = mX + b$

$b = mC_1 + C_2$

Age of the Earth





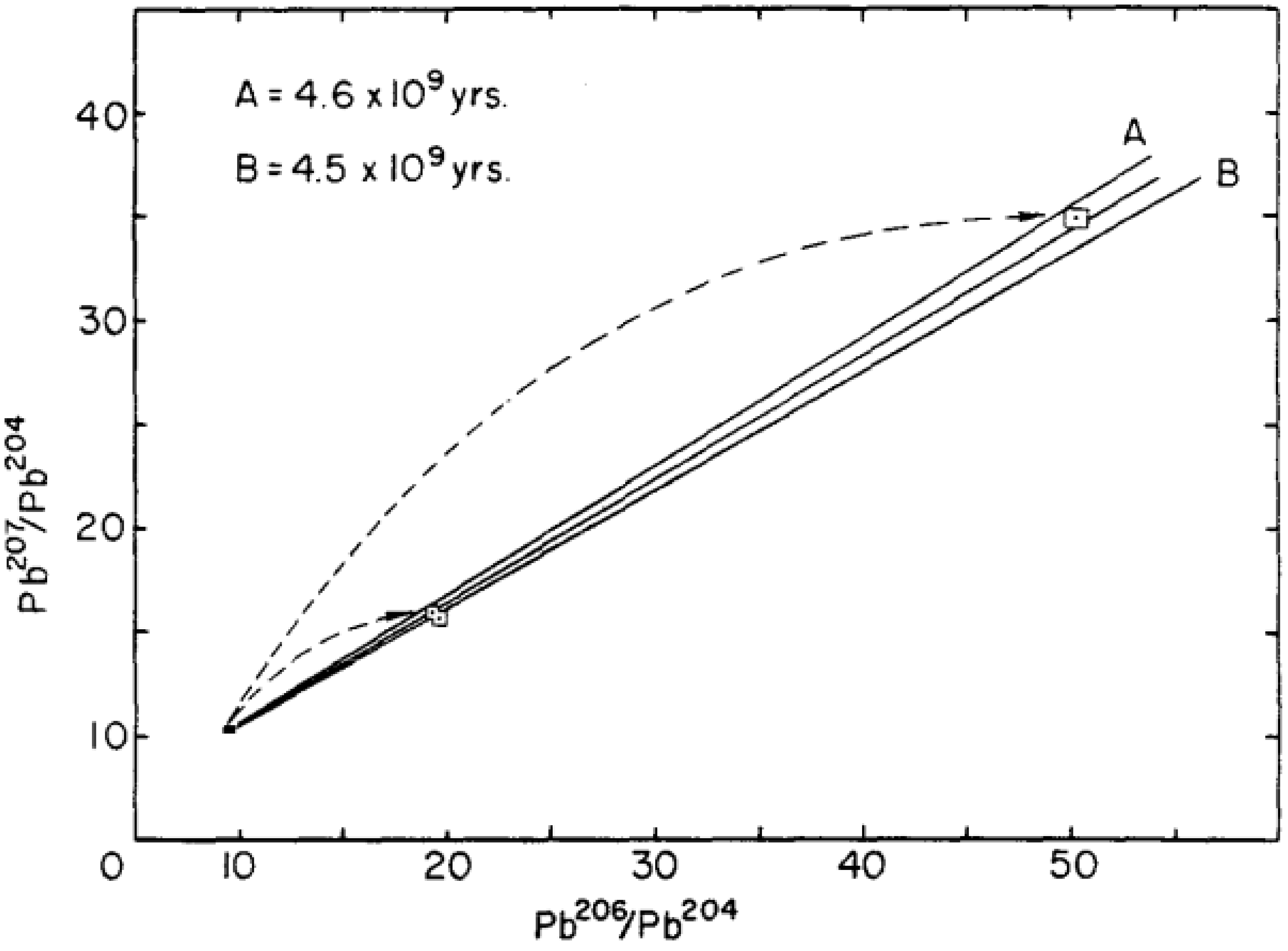


Fig. 1. The lead isochron for meteorites and its estimated limits. The outline around each point indicates measurement error.

Age of meteorites and the earth. Claire Patterson 1956