# EOS 423 // EOS 518

## Lab 1.1: Sedimentary Transport

Due: 1:30 pm January 27, 2023

You have one week to complete this assignment and upload your responses as a PDF to Brightspace. You are not excluded from working with others (pairs are recommended), but each person will submit their own copy of the assignment. **Responses to questions should be typed, using complete sentences and standard grammar.** Double check that your image resolution is high enough to read. If you write your response in a word processor, please export to .PDF before submitting your response.

A pair of students will be randomly selected on Monday to initiate and lead a discussion of the assignment. Be prepared to show your progress and discuss any challenges you still face.

| Question | 1 | 2 | 3 | 4 | 5 | 6 | 7 | **Total:** |
|---|---|---|---|---|---|---|---|---|
| Marks: | 2 | 2 | 2 | 2 | 4 | 13 | 2 | 27 |
| Score: | | | | | | | | |

In this lab, you will develop a model of the profile of a prograding delta. You will then use your model to understand the conditions that explain variations in delta shape.

# 1 Bulk sediment transport: diffusion

The change of elevation over time is proportional to the second partial derivative of the topography with respect to space (the curvature). This equation is sometimes referred to as the *hillslope application* of the **diffusion equation**:

$$\frac{\partial h}{\partial t} = K\frac{\partial^2 h}{\partial x^2} \qquad \text{(diffusion equation)}$$

The diffusion equation is derived from combining the **continuity equation**, which expresses the conservation of mass:

$$\frac{\partial h}{\partial t} = -\frac{\partial S}{\partial x} \qquad \text{(continuity equation)}$$

with an expression of sediment transport rate, $S$, as a **diffusive flux**. In this formulation, $S$ is linearly proportional to slope:

$$S = -K\frac{\partial h}{\partial x} \qquad \text{(diffusive flux)}$$

*Kenyon and Turcotte 1985* solves the diffusion equation analytically by introducing a coordinate system that moves along with the landward edge of the delta front, which is steadily prograding at a rate, $u_0$:

$$\xi = x - u_0 t$$

$$t' = t \qquad \text{(moving reference frame)}$$

and the associated partial derivatives:

$$\frac{\partial h}{\partial x} = \frac{\partial h}{\partial \xi}$$
$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial t'} - u_0\frac{\partial h}{\partial \xi}$$

Now, we can write the **diffusion equation within a moving reference frame** as:

$$\frac{\partial h}{\partial t'} - u_0\frac{\partial h}{\partial \xi} = K\frac{\partial^2 h}{\partial \xi^2}$$

In this reference frame, the height of the delta does not change with respect to time, so $\frac{\partial h}{\partial t'} = 0$, and this form of the diffusion equation reduces to the ordinary differential equation:

$$0 = \frac{d^2h}{d\xi^2} + \frac{u_0}{K}\frac{dh}{d\xi}$$

with the following solution:

$$h(\xi) = h_0 e^{\left(-\frac{u_0\xi}{K}\right)}$$

**Question 1** (2)

Describe (in words) the assumptions made about the flux of sediments on the upstream and downstream boundaries of this moving system.

**Question 2** (2)

The system above describes a delta prograding across a flat surface. If the modeled delta were to prograde down a ramp with a constant slope, what changes might you expect, if any, in the shape of the delta front?

**Question 3** (2)

Showing all your steps, derive the solution to the **diffusion equation within a moving reference frame** (a second-order, ordinary differential equation).

**Question 4** (2)

What does the following expression from *Kenyon and Turcotte (1985)* represent:

$$\frac{u_0}{K}(x - u_0t_0)$$

**Question 5** (4)

From equations to code. Using equations from *Kenyon and Turcotte (1985)* and a programming language of your choice:

(a) (2 points) Make plots that demonstrate delta progradation into a basin over four timesteps. *(hint: recreate the cartoon in Figure 14 from the paper with your code)*. For this question, assume some fixed diffusivity, progradation velocity and delta height above the basin floor.

(b) (2 points) Describe in a few short sentences how and why the height of the delta front changes with distance from the delta top.

**Question 6** (13)

Exploring variations in delta shape. Make plots that demonstrate:

(a) (2 points) How the shape of a delta front is a function of sediment diffusivity.

(b) (3 points) Using these plots, describe in a few short sentences how and why changes in sediment diffusivity influence bulk transport.

(c) (2 points) How the shape of a delta front is a function of progradation velocity.

(d) (3 points) Using these plots, describe in a few short sentences how and why changes in progradation rate influence bulk transport.

(e) (3 points) Under what conditions might *bulk sediment transport* (diffusion) lead to the steep foreset beds of the Gilbert-type delta? Include in your response discussion about how the sedimentary environment and sediment properties may control the diffusive coefficient (K).

**Question 7** (2)

Sedimentary Flux: The total volume of sediment added to a delta system during a time interval $\Delta t$ is illustrated in Figure 4 of *Kenyon and Turcotte (1985)*; i.e., the integrated area between the delta front at time $= t_0$ and the delta front at $t_1$. However, equation 25 in the paper gives a very simple equation for this quantity:

$$\text{volume} = \Delta t \cdot h \cdot u$$

where $h$ is the maximum height of the delta front and $u$ is the progradation velocity. Relying upon your work for **question 5a**, show numerically that equation 25 (above) from Kenyon and Turcotte is valid.