EOS 240: Lab Assignment 4

Chemical differentiation of the Earth

Due: 2:30 pm February 29, 2023 (Th section)
Due: 1:30 pm March 01, 2023 (F section)

You have **three** weeks to complete this assignment **(next week is your midterm and the following week is reading break)**. You should submit your response to the course Brightspace page as a single PDF file. **Additionally, we ask that you upload a copy of the scripts, code, or spreadsheets you used to complete the assignment. These documents will help us track down mistakes.** Responses to questions should be typed, using complete sentences and standard grammar. If you choose to support your answers with hand-drawn illustrations or hand-written calculations, you should scan or photograph the written work and integrate it into your PDF file as a figure. Double check that your image resolution is high enough to read. A google search of 'PDF combiner' will return a number of webpages that allow you to upload individual images and combine them into a single .pdf file (example: combinepdf.com). There are also a number of good apps for mobile phones. If you write your response in a word processor, please export to .PDF before submitting your response.

You are not excluded from working with others (pairs are recommended), but each person will submit their own copy of the assignment. In your submission, include the names of anyone you worked with on the assignment.

To answer the questions, you can perform calculations and make figures using Excel (an open source alternative: www.libreoffice.com), or with a program or programming language of choice.

Name : ________________________________

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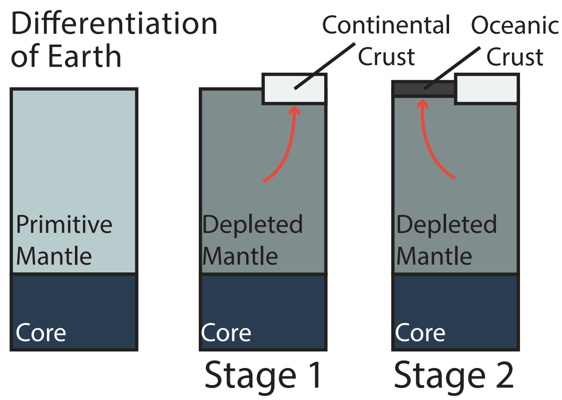
Marks:	10	13	2	25
Score:				

Introduction

In this lab, you will use a two stage melting model to extract the continental and oceanic crust from the primitive mantle of Earth. This greatly simplified model of *differentiation* is able to reproduce the elemental abundances of crustal rocks and offers quantitative constraints about the history of melt extraction from the mantle.

This simplified history begins with the primitive mantle (often referred to as the *bulk silicate earth*). This geochemical reservoir represents the mantle of Earth (after core formation) before the crust was extracted. In other words, the primitive mantle would be the result of could mix the entire crust back into the present day mantle. You will differentiate Earth in two discrete stages of melt extraction:



- Stage 1: A partial melt of the *primitive mantle* is extracted to form the continental crust, leaving behind a mantle that is depleted in highly incompatible elements.
- Stage 2: A partial melt of the left behind *depleted mantle* is extracted to form the oceanic crust.

You will use **Fractional melting** equations to complete these stages of differentiation, and you will directly compare your modeled melts to the average composition of continental crust and mid-ocean ridge basalt (oceanic crust).

The **partition coefficient**, D, describes the ratio of concentrations for an element between a mineral and a melt at equilibrium:

$$D = \frac{C_S}{C_L}$$

Fractional melting describes a scenario where some fraction of a rock melts and that melt is immediately separated from the rock. The concentration of a trace element in the remaining rock is described by this equation:

$$\frac{C_S}{C_0} = (1-F)^{(D^{-1}-1)}$$

The concentration of a trace element in an infinitely small fraction of melt, often referred to as the *instantaneous melt*, is described by this equation:

$$\frac{C_L}{C_0} = \frac{(1-F)^{(D^{-1}-1)}}{D}$$

C is the concentration of a trace element. Subscript L represents the melt phase. Subscript S represents the solid phase. C_0 means at the initial conditions when solid is 100% of the system. D is the partition coefficient, and F is the melt fraction (where 1 is 100% melt).

Question 1 (25)

Chemical differentiation of Earth

(a) (2 points) The *instantenous melt* in the equation above may pool or stall after physically separated from the melting rock. Derive an equation that describes the mean composition of this pooled melt, $\overline{C_L}$ (show your steps). *hint: you'll need to integrate* $\frac{C_L}{C_0}$ *from 0 to F. Also keep in mind that the change in melt fraction is equal to 1 minus the change in solid fraction,* $dF = -d(1-F)$.

(b) (1 point) If you have solved part *(a)* correctly, your derivation should maintain mass balance for any melt fraction:

$$C_0 = C_S(1-F) + \overline{C_L}F$$

Provide a figure to show that mass balance is maintained from $F = 0$ to $F = 1$. Your solution should be true for any positive D and C_0, but show the result for $D = 0.1$ and $C_0 = 1$.

(c) (2 points) You will now use this fractional melting model to differentiate the primitive mantle. The primitive mantle represents Earth after core formation but before extraction of the crust. Elemental abundances can be normalized to this primitive mantle starting point by taking a ratio of the observed concentration divided by the primitive mantle concentration. This normalization offers common reference frame for each element where a normalized concentration of 1 means that the composition is the same as the primitive mantle. Use your fractional melting model to extract 1% of the primitive mantle (where all initial normalized elemental abundances are 1). Make a figure showing the average composition of that melt ($\overline{C_L}$) and the residual mantle (C_S) for elements with bulk partition coefficients, D_{bulk} from 10^{-3} to 50. This figure should have log scale for both axes, and the x-axis should be the partition coefficient D from 10^{-3} to 50.

(d) (2 points) *lab4.csv* contains elemental abundances of average continental crust (**CC**), mid-ocean ridge basalt (**MORB**), and the primitive mantle (**PM**) in ppm (μg per g). Make a figure showing the continental crust and mid-ocean ridge basalt compositions normalized to the primitive mantle. The y-axis should be your normalized concentrations, and the x-axis should be populated by each element name. Sort your elements by decreasing abundance in the continental crust.

(e) (3 points) Discuss your figure from part *(d)* with at least one other person. Write down your collective observations about the similarities and differences between oceanic and continental crust compositions. If the continental crust and oceanic crust (mid-ocean ridge basalts) were both once part of the primitive mantle, can you explain how melting may have generated these two distinct chemical abundances? Write down your best guesses.

(f) (2 points) Assume that Rb, the most enriched trace element in the continental crust from your dataset, is infinitely incompatible (D_{bulk}=0). If the continental crust is a partial melt extracted from the primitive mantle, what melt fraction do you need to explain the observed Rb enrichment in the continental crust?

(g) (3 points) First, make a figure almost identical to your figure from part *(c)*, but use the melt fraction that you calculated in part *(f)*. Second, use this melt fraction F to solve for the bulk partition coefficient, D_{bulk}, of each of the elements in your data table. In other words, if the continental crust is a partial melt of the primitive mantle with a melt fraction of F, what bulk partition coefficient, D_{bulk}, does each element need to have to explain the observed concentration? Assume that these D_{bulk} values represent the partitioning of any melt extracted from the mantle (primitive and depleted) and use the partition coefficients to plot the continental crust and mid-ocean ridge basalt data on the figure. Use distinct symbols so that the each data set and the modeled values are distinct. Previously in part *(f)* we assumed D_{bulk} of Rb was 0, but to avoid division by zero errors you can set Rb to 1e-3.

(h) (3 points) The extraction of the continental crust has left the remaining mantle depleted in highly incompatible elements (your C_S). If mid-ocean ridge basalts are partial melts of this depleted mantle, what F of melt best explains the mid-ocean ridge basalt data? In last weeks lab you used *mean squared error* to find the best fitting parameters to a model.

$$mse = \frac{1}{n}\sum_{i=1}^{n}(O-E)^2$$

Use the same approach here to determine the best fitting F. Support you answer with a figure that shows the misfits between your MORB data and predicted MORB data for melt fractions from 0 to 0.2. Your X-axis should be the melt fraction, F, used in each *mean squared error* calculation, and the Y-axis should be the *mean squared error. A log scale for the Y-axis can be helpful.*

(i) (1 point) Add your best fitting melt of the depleted mantle to your figure from part *(g)*. Describe where this model succeeeds and where this model fails to match the mid-ocean ridge data.

(j) (4 points) You can improve your model-data fit in part *(i)* by recycling a small fraction of the continental crust back into the depleted mantle before extracting mid-ocean ridge basalt. Determine how much continental crust you need to recycle to explain the data, and add this new modeled melt to your figure from part *(g)*. Support you modeling choices with an analysis of the misfit between your observations and model.

(k) (2 points) Use the provided data and your analysis of that data to justify the following statement:

> The continental crust has depleted a large part of the mantle in many incompatible trace elements, and this depleted part of the mantle is now the source of oceanic crust.
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> —Albrecht W. Hofmann, 1988