EOS 423 // EOS 518

Lab 1.3: Sedimentary Transport (Applying the Numerical Model)

Due: 1:30 pm February 14, 2025

You have two weeks to complete this assignment and upload your responses as a PDF to Brightspace. You are not excluded from working with others (pairs are recommended), but each person will submit their own copy of the assignment. **Responses to questions should be typed, using complete sentences and standard grammar.** Double check that your image resolution is high enough to read. If you write your response in a word processor, please export to .PDF before submitting your response.

A pair of students will be randomly selected on each Monday to initiate and lead a discussion of the assignment. Be prepared to show your progress and discuss any challenges you still face.

Note: because assignment 1.3 is a two-week assignment, it will be worth twice as much as assignment 1.1 and 1.2.

Question	1	2	3	**Total:**
Marks:	7	13	26	46
Score:				

Building stratigraphic records with matrix math

In part **1.1** of this module, we learned about how in many instances, delta morphology could be shown to be controlled by bulk sediment transport. Mathematically, this claim means that *diffusion* is a dominant process in building deltas, where the change of elevation over time is proportional to the second partial derivative of the topography with respect to space (the curvature). This equation is sometimes referred to as the *hillslope application* of the **diffusion equation**:

$$\frac{\partial h}{\partial t} = K \frac{\partial^2 h}{\partial x^2} \tag{diffusion equation}$$

We first looked at steady state solutions of the diffusion equation (i.e., where $\frac{\partial h}{\partial t} = 0$, through use of a moving reference frame). This allowed the derivation of an *analytical* solution to the diffusion equation, and could be used to predict the bathymetry of deltas whose morphology was not changing with time.

Of course, having tools to predict how delta morphology and coastline bathymetry can *change* as conditions change, such as from variations in sediment type, relative sea level or sediment supply, would be very useful. In studies of the sedimentary rock record and the history it tells, these types of boundary conditions frequently change. This need was the motivation for part **1.2**, where you built a numerical model of bulk transport using an *implicit* finite difference scheme called the Crank-Nicolson algorithm.

This approach estimates how topography changes with time by taking the average of the central-difference estimate of the second partial derivative of the topography with respect to space at both the current time step and the future timestep:

$$h_i^{t+\Delta t} - h_i^t = r\ (h_{i-\Delta x}^t - 2\ h_i^t + h_{i+\Delta x}^t + h_{i-\Delta x}^{t+\Delta t} - 2\ h_i^{t+\Delta t} + h_{i+\Delta x}^{t+\Delta t})$$

with r known as the Fourier number:

$$r = K \frac{\Delta t}{2\ \Delta x^2} \tag{Fourier number}$$

This equation forms a system of linear equations that can be rearranged and solved using linear algebra:

$$\begin{bmatrix} 2r+1 & -r & 0 & \\ -r & 2r+1 & -r & \\ 0 & -r & 2r+1 & \end{bmatrix} \begin{bmatrix} h_0^{t+\Delta t} \\ h_i^{t+\Delta t} \\ h_N^{t+\Delta t} \end{bmatrix} = \begin{bmatrix} 1-2r & r & 0 & \\ r & 1-2r & r & \\ 0 & r & 1-2r & \end{bmatrix} \begin{bmatrix} h_0 \\ h_i \\ h_N \end{bmatrix} + 2r \begin{bmatrix} b_0 \\ b_i \\ b_N \end{bmatrix} \tag{1}$$

If we call the matrices on the right-hand side and left-hand side A and B, respectively, we can rewrite the expression as:

$$Ah^{t+\Delta t} = Bh^t + b^t$$

where A and B are square matrices with whose side-length is equal to the length of h and b is a vector of boundary conditions. The first and last entries of b define the boundary conditions of your model space - setting these will fix the fluxes into the left and right edges of the system, and thus set the topography at those edges. The matrix equation above is solved by multiplying both sides by A^{-1}:

$$h^{t+\Delta t} = A^{-1}(Bh^t + b^t)$$

Now you have the tools to solve the diffusion equation numerically. In each time step, the current topography (h^t) and boundary conditions (b^t) can be used to predict the topography in the next time step ($h^{t+\Delta t}$). This new topography, in turn, will be used to predict the next time step, and so on, until your model run ends. One interesting way to to use this model is to introduce a prescribed sedimentation term, represented here by the vector s:

$$h^{t+\Delta t} = A^{-1}(Bh^t + b^t + s^t)$$

The vector s can be used to add topography to some part of your model space, and then allow that added topography to be modified through diffusion. For example, if you wish to simulate delta growth, a sediment flux term (which you define) could be added where topography intersects local sea level (which you also define). If this intersection occurs at h_i, then s_i would become non-zero to reflect a growth in topography. As topography changes, so potentially can the location of sediment delivery.

As the old adage goes, *"All models are wrong, but some models are useful."* Models will *never* capture every single biological, chemical or physical process at play in the natural world that may be having an influence on the surface process you are studying.[1] Rather, models should be designed to capture some aspect of how the natural world works, and model experiments should be run to see how it responds to different parameter values or forcings. Although it is very simple, the numerical model you have developed for bulk sediment transport is rooted in some well-founded assumptions about how sediment is transported. Namely, that sediment fluxes should behave as *diffusive fluxes,* acting to smooth topography over time.

Thus, the model can be used to explore how topography changes under various boundary and initial conditions, and compared to real world data (for example, lithology logs from stratigraphic sections, or 2-D seismic data, measured across a shallow-to-deep transect of an ancient continental margin). If the model captures important aspects of the data, you can potentially make inferences about the processes recorded by your stratigraphic records (i.e., relative sea level rose, sediment flux increased, etc.). If the model fails to explain the data in some important way, this result suggests important processes are not currently captured by your model, and may spur more model development. Either outcome would be useful to science!

So, let's get some practice using this model, examining the results, and thinking about what those results mean...

[1] Another good saying goes, *"The best model of a cat is a cat."*

Questions

Question 1 (7)

Experiments with your model

In question 2 from assignment 1.2, you ran a model experiment that explored how topography changes under constant sediment flux and constant sea level. The model began with an equilibrium topographic profile (i.e., a straight line between fixed heights at the left and right margins of your model space). The produced topographic profile, at various time snapshots in the model run, looked something like Figure 1 below. There are two notable things about these results:

1. A broad, flat shelf, with a height equal to local sea level, quickly develops. Beginning at the most basin-ward (in this case, right-ward) point on this shelf, there is a dramatic drop in topography as one moves further to the right in the model space.
2. Let's call this point the shoreline (marked as dots in Figure 1), since it is the most basin-ward point where topography is at sealevel. It is notable that this point is moving *right-ward*. This behavior is called *progradation*, as sediment gradually fills in available accommodation space. Contrasting behaviors could be that the shoreline *retogrades* and moves landward with time, or *aggrades* and stays at the same point (on the x-axis, at least) with time.

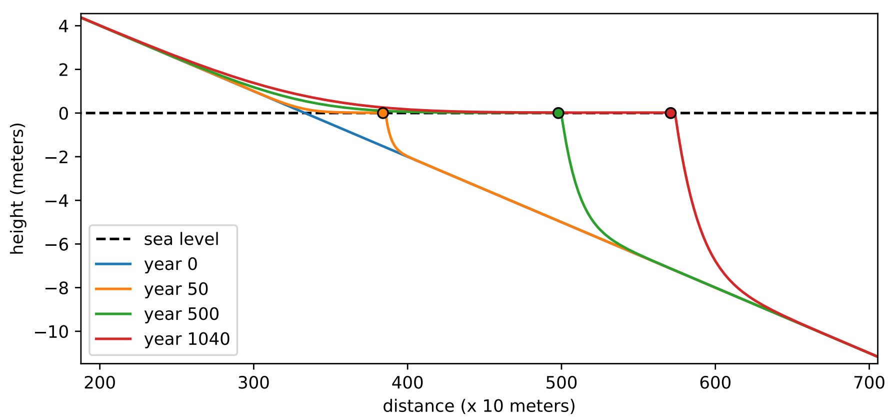

Figure 1: an answer to question 2 from assigment 1.2

(a) (2 points) Thinking in terms of depositional settings (e.g., alluvial fan, deep marine, etc.), describe the different environments encountered as you move from point 200 to point 700 on the year 1,040 (red) line.

(b) (2 points) It turns out that the development of a flat shelf is very common feature in model runs such as these. Why does a flat shelf invariably develop? *Hint: run a model experiment where your initial topography is flat or essentially flat, as opposed to an angled slope as in question 1.2.2. If you do this, keep constant the x-point where sediment is added in every time step (say, always add at the 500th x-grid point).*

(c) (3 points) For a given diffusivity, demonstrate how variations in sediment flux can lead to a prograding, aggrading and retrograding shoreline. Use the *same* sea level history for each experiment. Make a plot for each scenario (three total), as well as providing the model parameters used. In each plot, height will be on the y-axis and distance will be on the x-axis (like in Figure 1). Plot enough topographic lines at different time steps to clearly show the temporal trend in the shoreline (retrogradation, aggradation and progradation).

Question 2 (13)

Analyzing model results

For this next set of questions, you will analyze a set of model results (found in `model_results.csv`), without fully knowing the parameters or boundary conditions used to run the model. In the CSV file, the first column is the model time (in years), and the following 1000 columns are topography (in meters) at each x-axis grid point for the corresponding time step. Spacing between these grid points is 10 meters. If you are using Python, you can use the following code to import the data, after making sure the file is in your working directory:

```
----------
import csv

#empty lists for data
model_time=[]
model_topo=[]

#open the data file
with open("model_results.csv","r") as fid:
    data = csv.reader(fid, delimiter=",")

    #reads the data in line-by-line
    for row in data:
        model_time.append(float(row[0]))
        model_topo.append([float(r) for r in row[1:]])
----------
```

Here, `model_time` is a list and `model_topo` is a list of lists. For example, `model_time[100] = 990.1`, and `model_topo[100]` is the topography for that time step. If you want to see how topography changed over the course of the model run for the 500^{th} grid point, you would write `model_topo[-1][500] - model_topo[0][500]` = 2.85 meters.

(a) (3 points) Make a plot of the horizontal position of the shoreline (on the y-axis) versus time (on the x-axis). What does the temporal trend of the shoreline tell you about the sea level history used to produce this model? *Hint: there is no set way to determine the shoreline, so you will need to experiment. The function* `numpy.gradient` *might be helpful. When you have a method that you think is working, check it visually - i.e., plot the topography for a handful of times, and estimate where the shoreline is by eye. How does this manual determination compare with your programmatic determination for shoreline location?*

(b) (1 point) Base on the position of the coastline at the beginning and end of the model run, what was the direction and total amount of sea level change in this simulation?

(c) (2 points) For regions of the x-grid that accumulated sediment (so final topography - initial topography is >0), what is the mean value and variability of total NET accumulation of sediment? Show this distribution as a histogram plot (see `https://www.datacamp.com/community/tutorials/histograms-matplotlib` if you want a refresher on histograms, or how to make them in Python). How does this distribution compare to the total sea level change you determined in question 2b?

(d) (3 points) For each the grid point that accumulated sediment, what percent of the total elapsed model time is represented by NET sedimentation (i.e., a gain in height that was not later lost to erosion) versus hiatus or erosion? Make a plot to show this result, with the x-axis being the x-grid position where sediment accumulated and the y-axis being the percent of total time represented by net sedimentation. Where is the most complete sedimentary rock record (i.e., where the most time is represented by rock), and where are there the most gaps? *Hint: examine and think about Figure 2 before starting this task.*

(e) (1 point) For the grid points where the net accumulation of sediment is relatively high (at least ≥ 10 meters of topography gain), calculate the apparent sediment accumulation rate by dividing the total NET thickness by total time elapsed.

(f) (3 points) Now calculate apparent sedimentation rates with a moving window moving up through your selected sections (say, in 3 meter thick intervals throughout each section). How do these calculated sedimentation rates compare to your single "full section" determinations? Take the base 10 logarithm of the sedimentation rates determined in (e) and (f), and plot the distribution of these log-transformed values as a histogram (e.g., the latter in red bars, and the former in blue, so they can be easily distinguished).

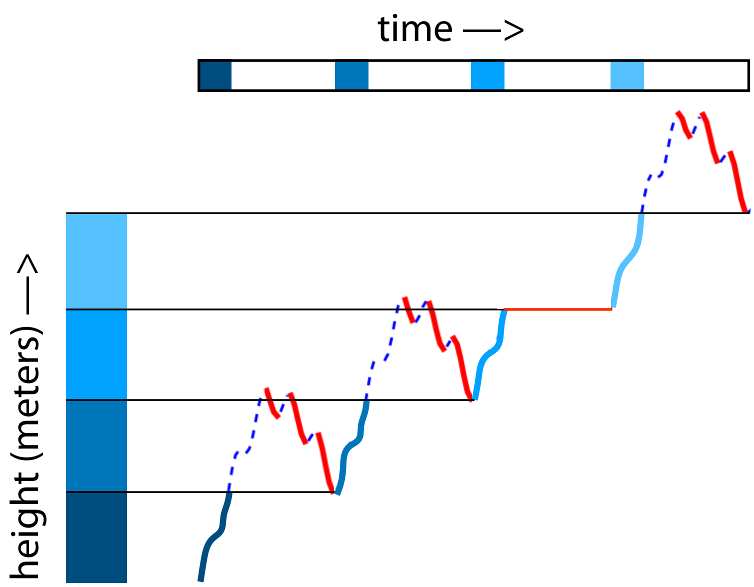

Figure 2: The wiggly curve is a cartoon showing how height (y-axis) changes with time (x-axis) at a particular point in the Arrakis Basin. Dashed blue lines represents time periods when topography grew but was later destroyed (shown as the downward red trajectories). The far left shows the NET rock record left behind, from which you could determine apparent sedimentation rates by measuring some thickness (Δh) and dividing it by the total elapsed time (Δt) represented by that thickness.

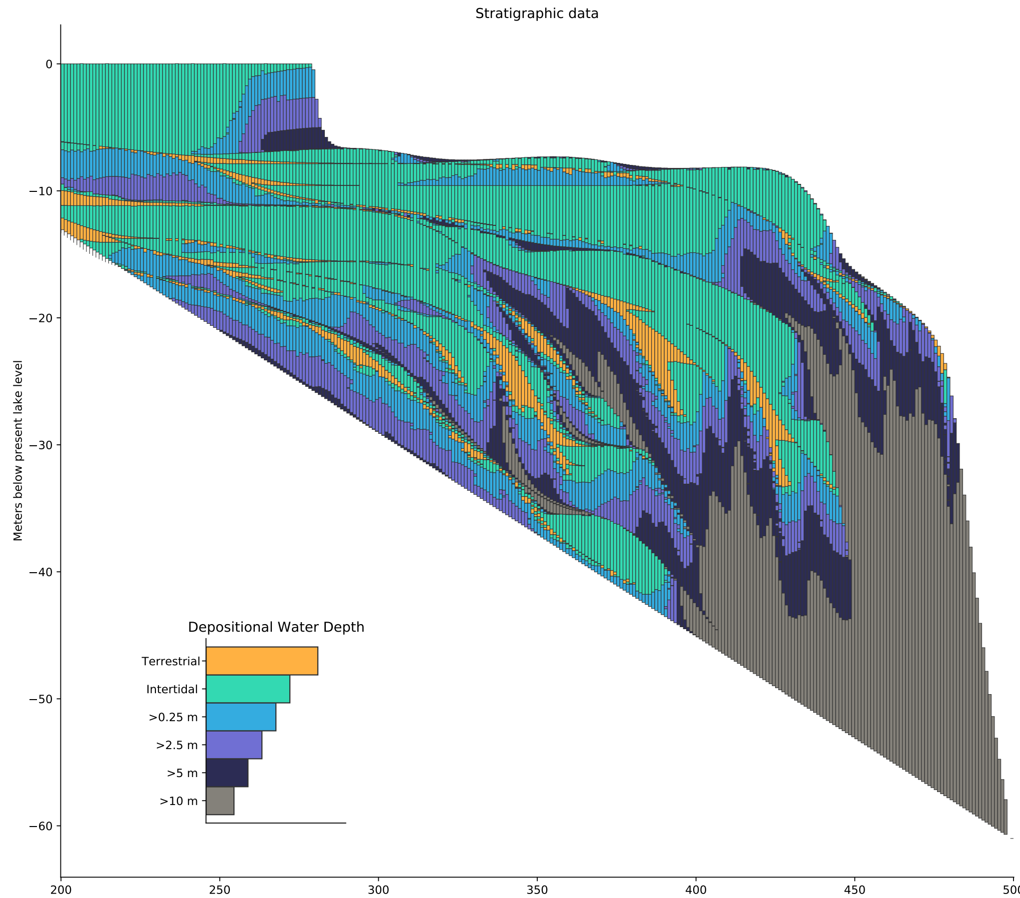

Figure 3

Question 3 (26)

Reconstructing Past Sea Level Change

Figure 3 contains stratigraphic data you have collected from the delta of the Redwater river in the Sea of Rhûn. The Redwater river was redirected to this basin 400,000 years ago by rapid tectonic shifts. The river has always delivered a constant 14 m^2 of sediment per year to the basin. However, sea level has changed drastically in the basin over this time (the basin is also gradually subsiding). Your task is to reconstruct the sea level history of the Sea of Rhûn from the stratigraphic data you have collected. The stratigraphic columns are plotted at their current height relative to sea level today.

(a) (3 points) Where do you think the edge of the delta is today? Describe the accommodation space landward and basinward from this point.

(b) (2 points) How does the depositional environment change vertically and laterally in this basin? If you recognize any general patterns or trends, what processes are responsible for those trends?

(c) (2 points) How does the stratigraphic thickness change laterally in this basin? Describe why the thicknesses change across the basin.

In your response to the following questions, please provide an annotated version of Figure 3.

(d) (3 points) Identify *flooding surfaces* in each core. A *flooding surface* is defined as a sharp contact that separates overlying younger strata with deep-water facies from underlying older strata with shallow-water facies.

(e) (3 points) To the best of your ability, correlate the flooding surfaces across each stratigraphic column. You will be unable to correlate all of the surfaces into all of the cores. Why?

(f) (3 points) For each correlated flooding surface, estimate the sea level change associated with that surface.

(g) (3 points) Is sea level in the basin higher today than 400 kya? If so, by how much?

(h) (7 points) Describe the history of sea level in this basin (the number of changes, and the magnitude of each change). Use your observations documented above to support your arguments. *(Bonus 4 points: run your own model with your preferred sea level curve and identify and discuss the reasons for any differences in your basin when compared to the dataset above. Your model output must be at least similar to the dataset above to get bonus credit here.)*