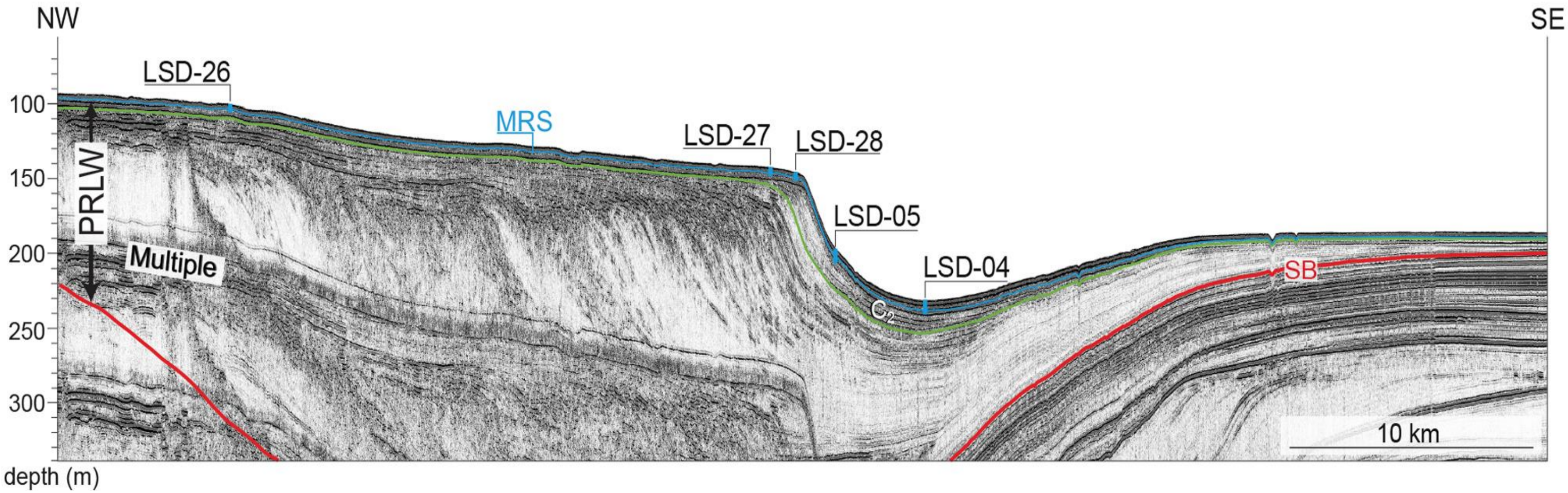
Lecture 2: Simplifying Surface Transport

1. Motivation: the stratigraphic record
    - An example
    - Modeling basin fill
2. What is diffusion?
    - Numerical example
    - Diffusive flux: what is it in *words* and *math*
    - Continuity equation: what is it in *words* and *math*
    - Diffusion as a transport mechanism
3. Assignment 1: diffusive transport

We acknowledge and respect the lək̓ʷəŋən peoples on whose traditional territory the university stands and the Songhees, Esquimalt and W̱SÁNEĆ peoples whose historical relationships with the land continue to this day.

Po River (Italy)



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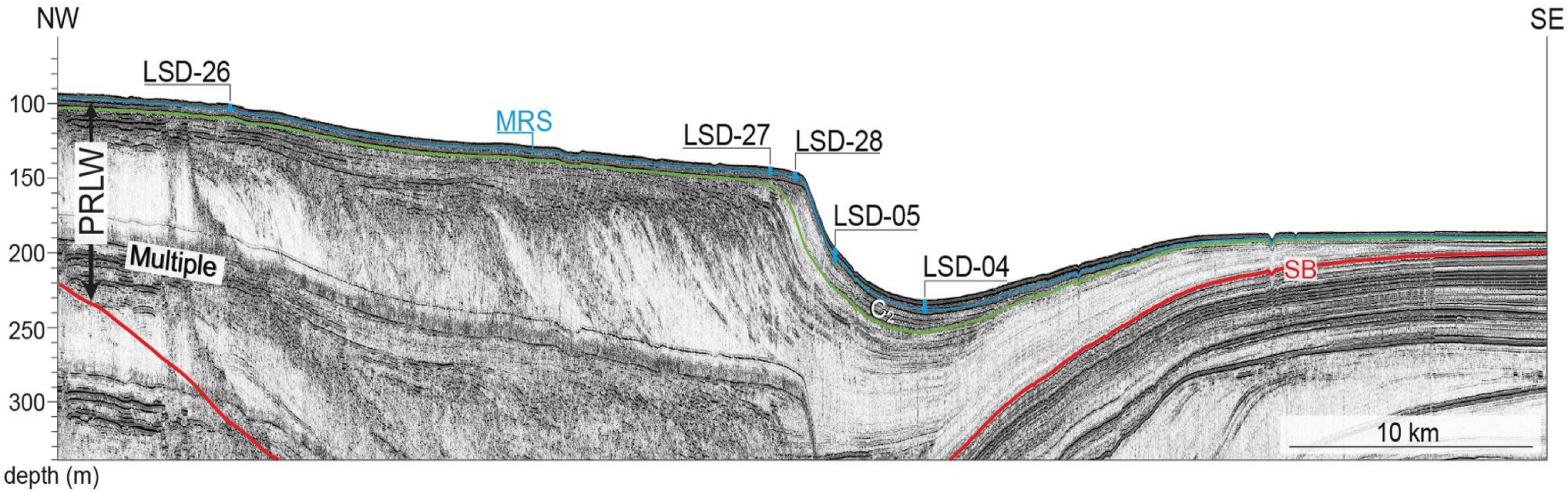
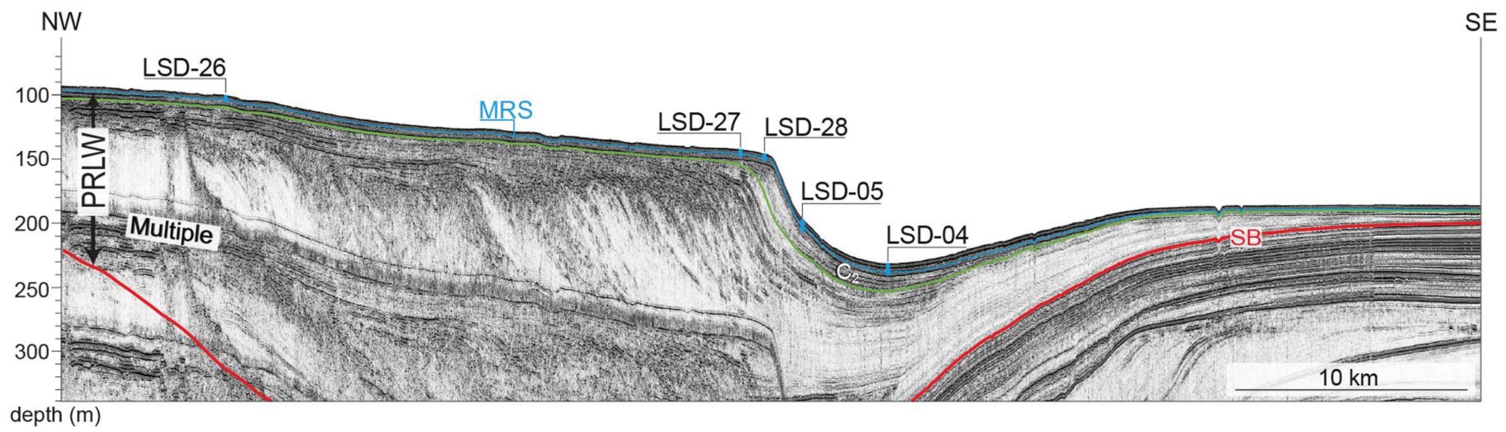


What do you see? Are there any patterns or features that stand out?

Po River (Italy)

Class observations/interpretations



- topsets (flat lying reflectors behind foresets) seem to indicate local sea level rise
- foresets prograde (move into the basin) and maybe increase a little bit in elevation

Simulating basin fill

Forward models can be used to explore some of these complex interactions. Generally there are two types of numerical models:

- complex fluid-flow models (think about the complexity of coastal dynamics)
  - use very real physical equations to simulate movement in 3-D of each grain
  - need accurate and high resolution data
  - computationally expensive
  - challenging to learn from or compare to stratigraphic data

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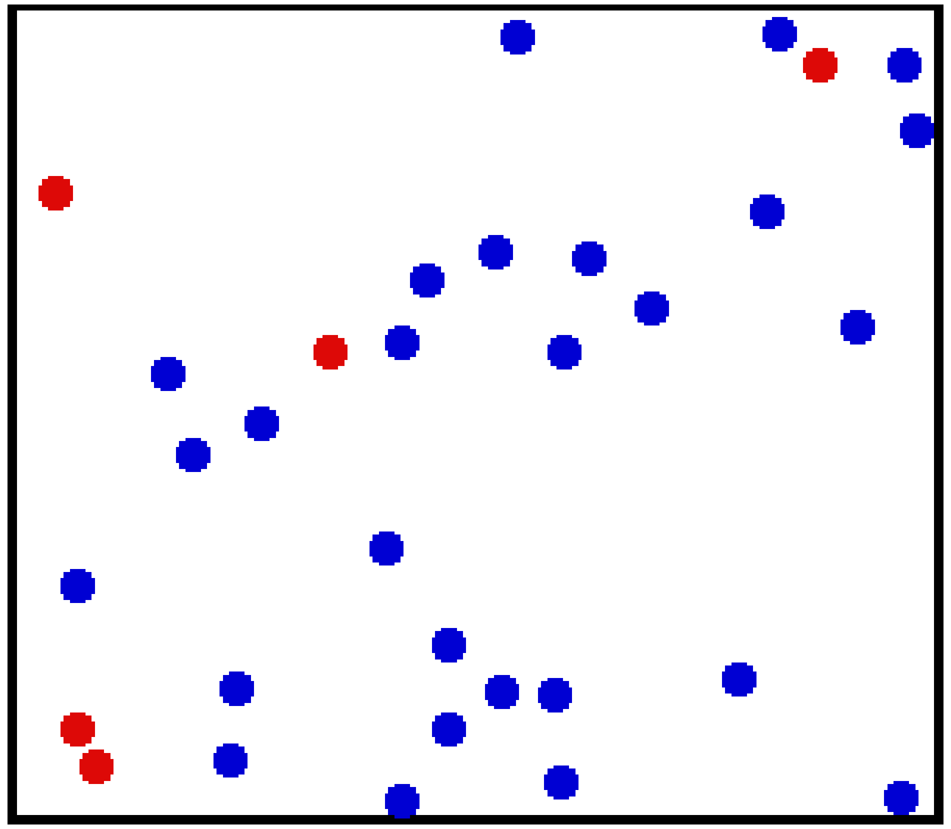
  - use very real physical equations to simulate movement in 3-D of each grain
  - need accurate and high resolution data
  - computationally expensive
  - challenging to learn from or compare to stratigraphic data
- empirical transport models
  - approximate transport laws combined with conservation equations (**diffusion**)
  - does not mimic in detail each geologic process
  - computationally inexpensive
  - model outputs can be easily compared to real data

What is diffusion?

Lets take a look at some examples of diffusion..

A working definition of diffusion: *is the net movement of anything (for example, atom, sediments, energy) from a region of higher concentration to a region of lower concentration.*



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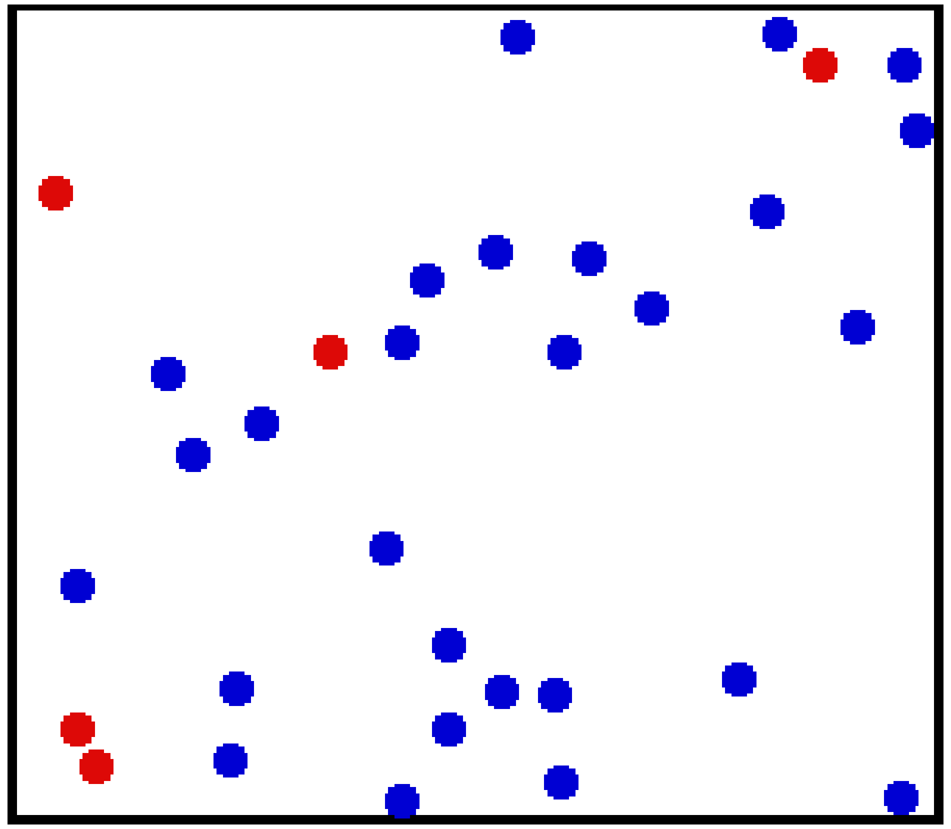

Note that it is the complex interaction of particles above that causes diffusion, not some inherent desire to randomly move.

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Diffusion is a good predictor of the **net effect** many transport related processes such as tidal currents, storms, bioturbation, the growth and death of reefs, etc.

- acts to decrease topographic gradients (gravitational potential energy)
- useful for building intuition about the stratigraphic record

The key here is that we can estimate the net change without understanding the details of every interaction.

Numerical example

If diffusion is the result of complex interactions, we can simulate those processes with random motion, and explore the system with a simple script. First, let's create a 2D world of 0's and 1's and introduce random motion.

Import some tools

In [4]:

```
import numpy as np
from matplotlib import pyplot as plt
from tqdm.notebook import tqdm
import seaborn as sns
sns.set_context('talk')
%matplotlib inline
```

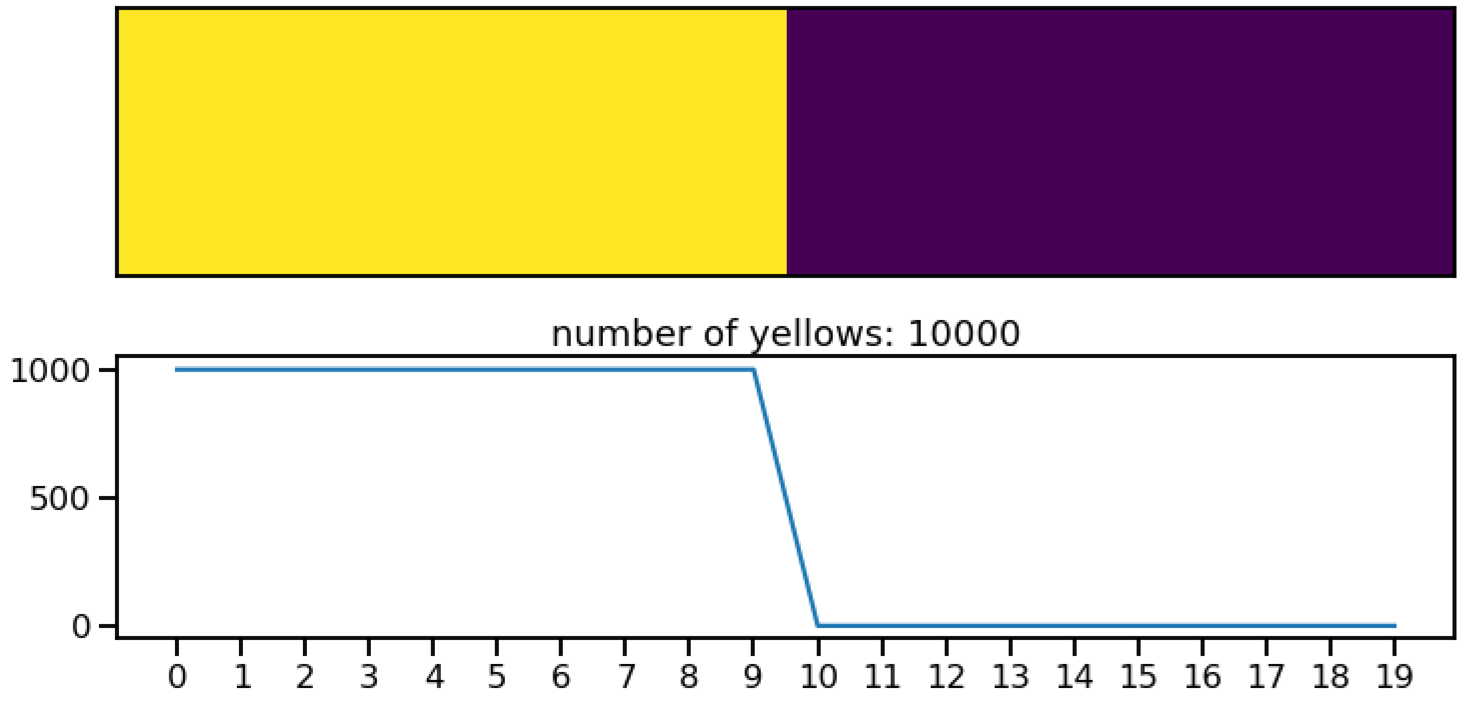
In [5]:

```python
row=[True]*10+[False]*10
row=[row]*1000
row=np.array(row)
print(row)
new_row = np.copy(row)
```

```
[[ True  True  True ... False False False]
 [ True  True  True ... False False False]
 [ True  True  True ... False False False]
 ...
 [ True  True  True ... False False False]
 [ True  True  True ... False False False]
 [ True  True  True ... False False False]]
```

In [6]:

```python
fig=plt.figure(figsize=(12,6))
plt.subplot(2,1,1)
plt.imshow(row,interpolation='nearest')
plt.gca().set_aspect(1/250)
plt.gca().set_xticks([])
plt.gca().set_yticks([])
plt.subplot(2,1,2)
plt.plot(np.arange(20),np.sum(row,axis=0))
plt.gca().set_xticks(np.arange(20))
plt.gca().set_title('number of yellows: %2.0f' % (np.sum(row)))
plt.gca().set_aspect(1/250)
```



Introduce *random* motion

In [7]:

```
for n in range(10):
    row=np.copy(new_row)
    for i in range(1,1000-1):
        for j in range(1,20-1):
            change_x = np.random.choice([-1,0,1])
            change_y = np.random.choice([-1,0,1])
            new_row[i,j] = row[i+change_x,j+change_y]
```

In [8]:

```
def make_fig(row,show_grad=False):
    fig=plt.figure(figsize=(12,6))
    plt.subplot(2,1,1)
    plt.imshow(row,interpolation='nearest')
    plt.gca().set_aspect(1/250)
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    plt.subplot(2,1,2)
    plt.plot(np.arange(20),np.sum(row,axis=0))
    plt.gca().set_xticks(np.arange(20))
    plt.gca().set_title('%% change in yellows: %2.2f' % (100*np.sum(row)/10000-100))
    plt.gca().set_aspect(1/250)
    if show_grad:
        plt.gca().twinx().plot(np.arange(1,21),np.gradient(np.sum(row,axis=0)),color='r',label='gradient')
        plt.gca().legend(loc='best')

make_fig(row)
```

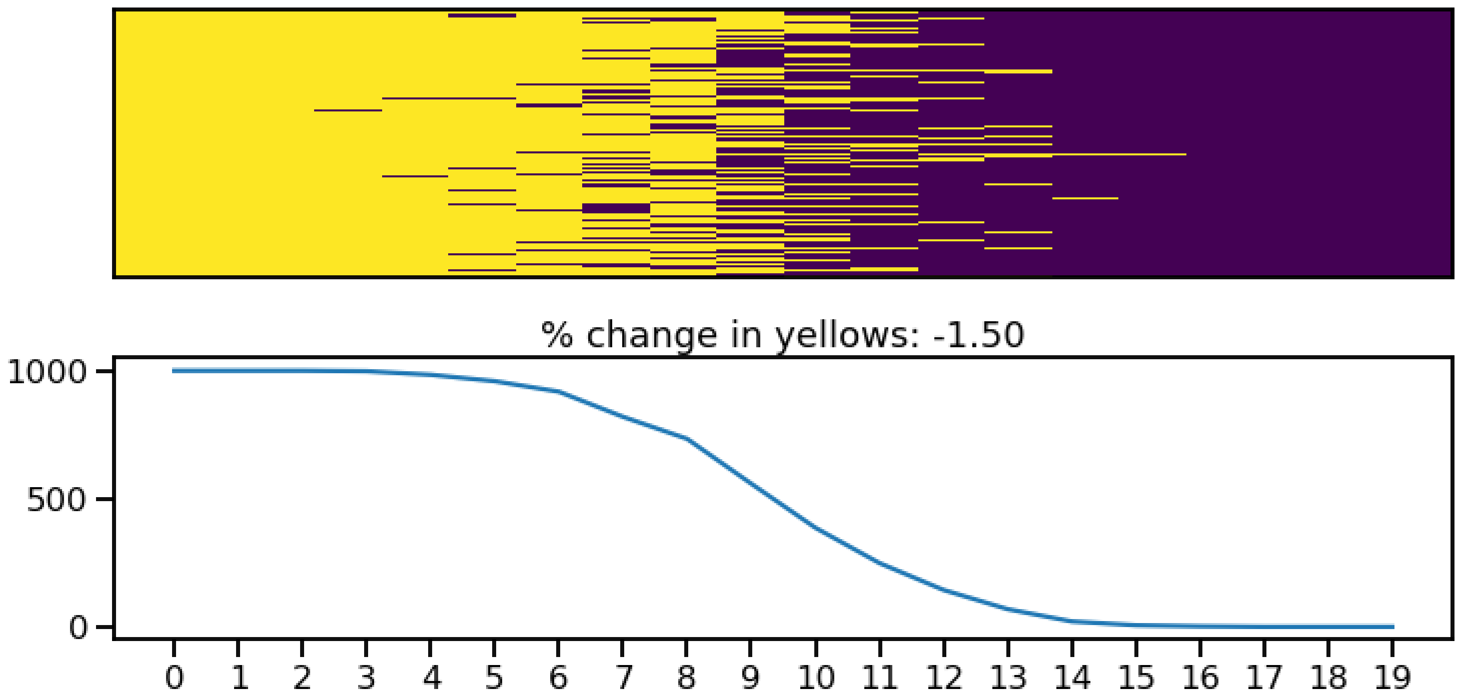

So how do we describe the change in the amount of yellow?

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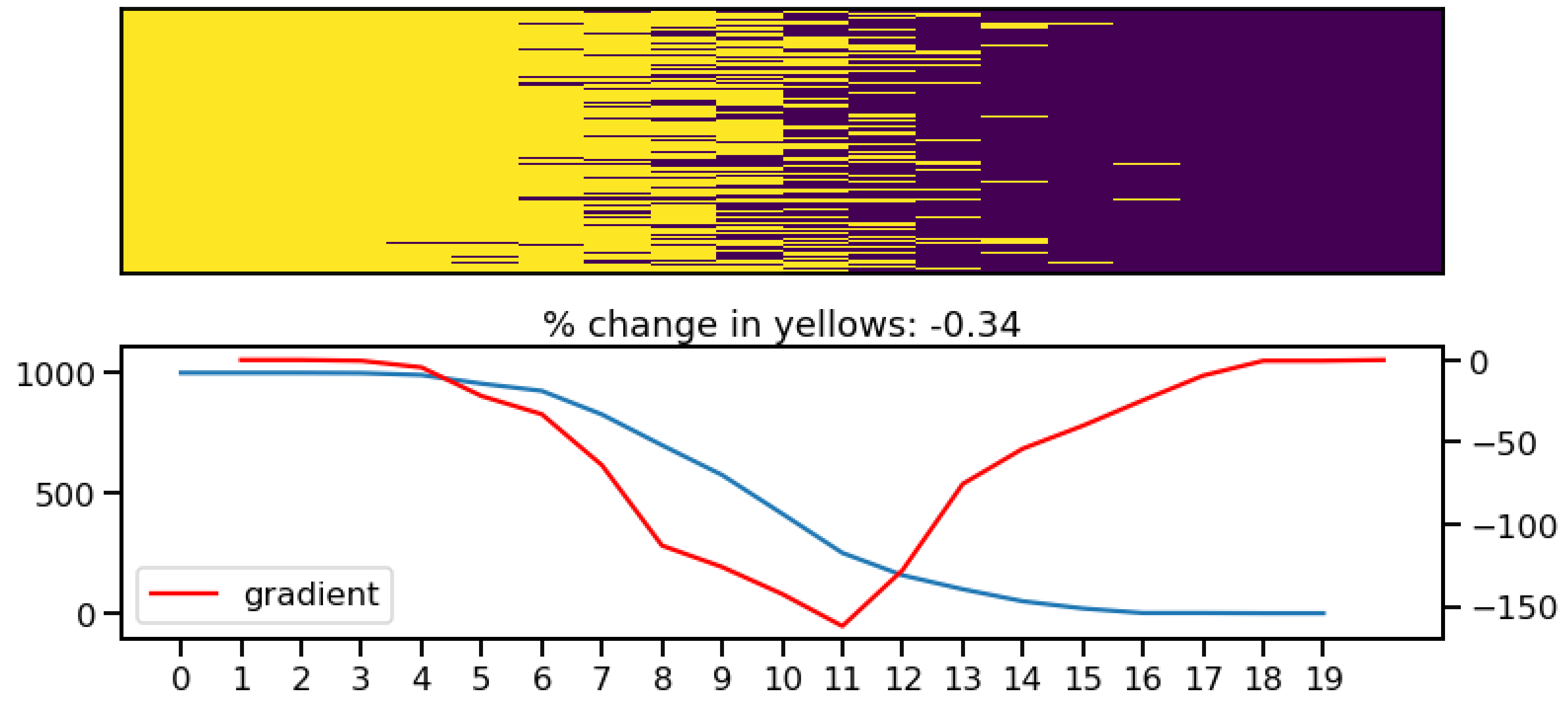

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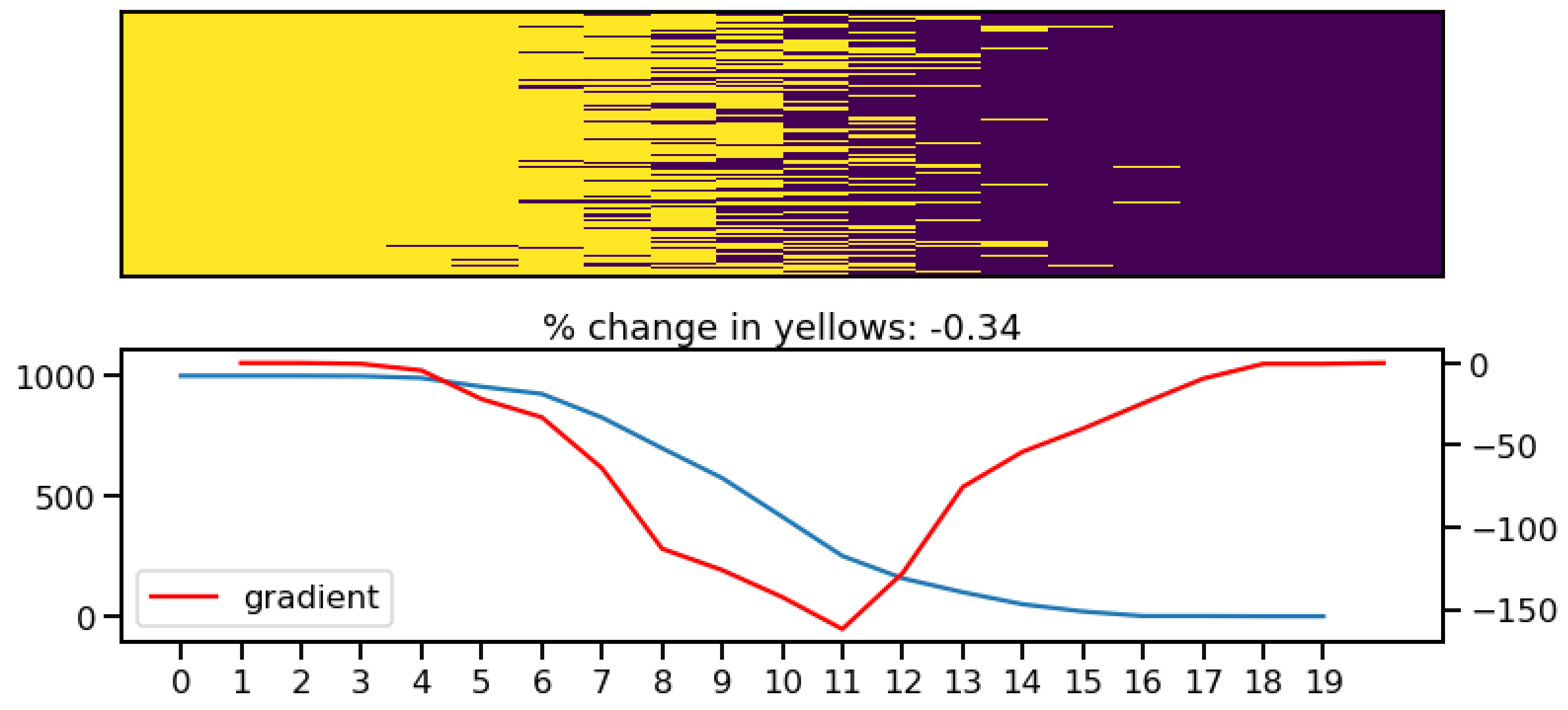

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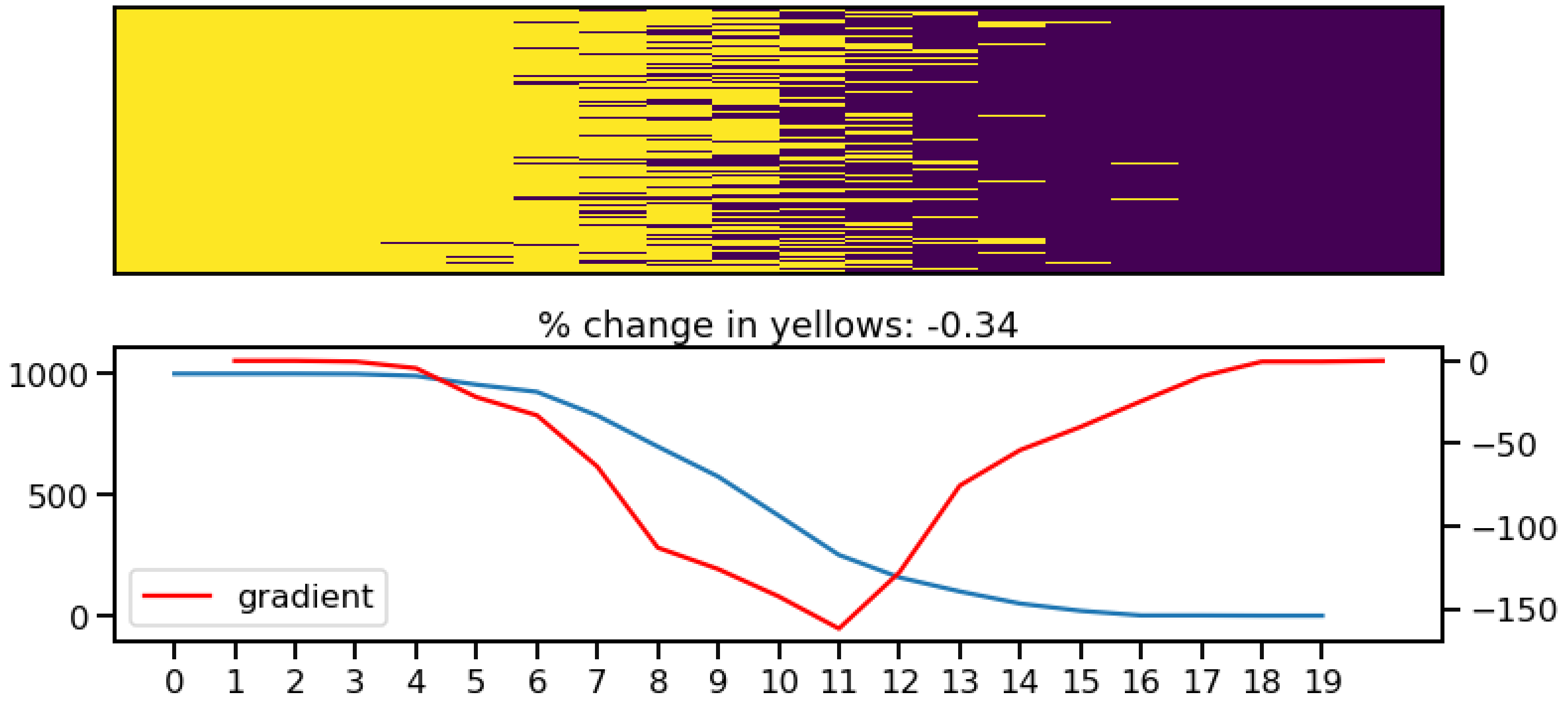

where is the diffusive flux largest?

- Flux is largest where the gradient is largest (they are proportional)

Diffusive flux: what is it in *words*

Lets consider a property, u, that is distributed over some spatial grid, x. We are interested in what happens to u when we introduce random transport (**diffusion**).

- Diffusion occurs when a (1) conservative property (u, like mass, energy, momentum) moves through space (2) at a rate proportional to a gradient

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- [tranportation rate of u] is proportional to [change in amount of u over some distance]
- [the flux Q_u] is proportional to [the spatial gradient of u]

Diffusive flux: what it is in *math*

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why is there a negative sign?

- the flux constant is negative to indicate a transfer from regions of higher u values to lower

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- Now, let's tackle **point number one** (mass balance):
  - [change in u with time] is equal to [change in transport rate of u with distance]
  - [rate of change in u] is equal to [the flux gradient of u]

Continuity equation: what is it in *math*

This equality is known as the **continuity equation** (conservation of mass/energy/momentum):

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Or in proper mathematical terms:

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How is this equation a conservation of mass?

By combining these two equations, we can show u will change over time as a function of the second spatial derivative of u (*the curvature*):

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Applying the diffusion equation to a physical system has two general requirements:

Diffusion as a transport mechanism

When we assume that **diffusive transport** exerts an important control on landscape dynamics we generally accept that particles on the surface of the planet are in constant complex motion, and there is some **downslope bias introduced by gravity** to those motions. Over time, **diffusion** can be assumed to smooth topography (by the transport of material downslope).

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Note: we wouldn't want to use this model to explain transport in rivers, why not?

A look ahead

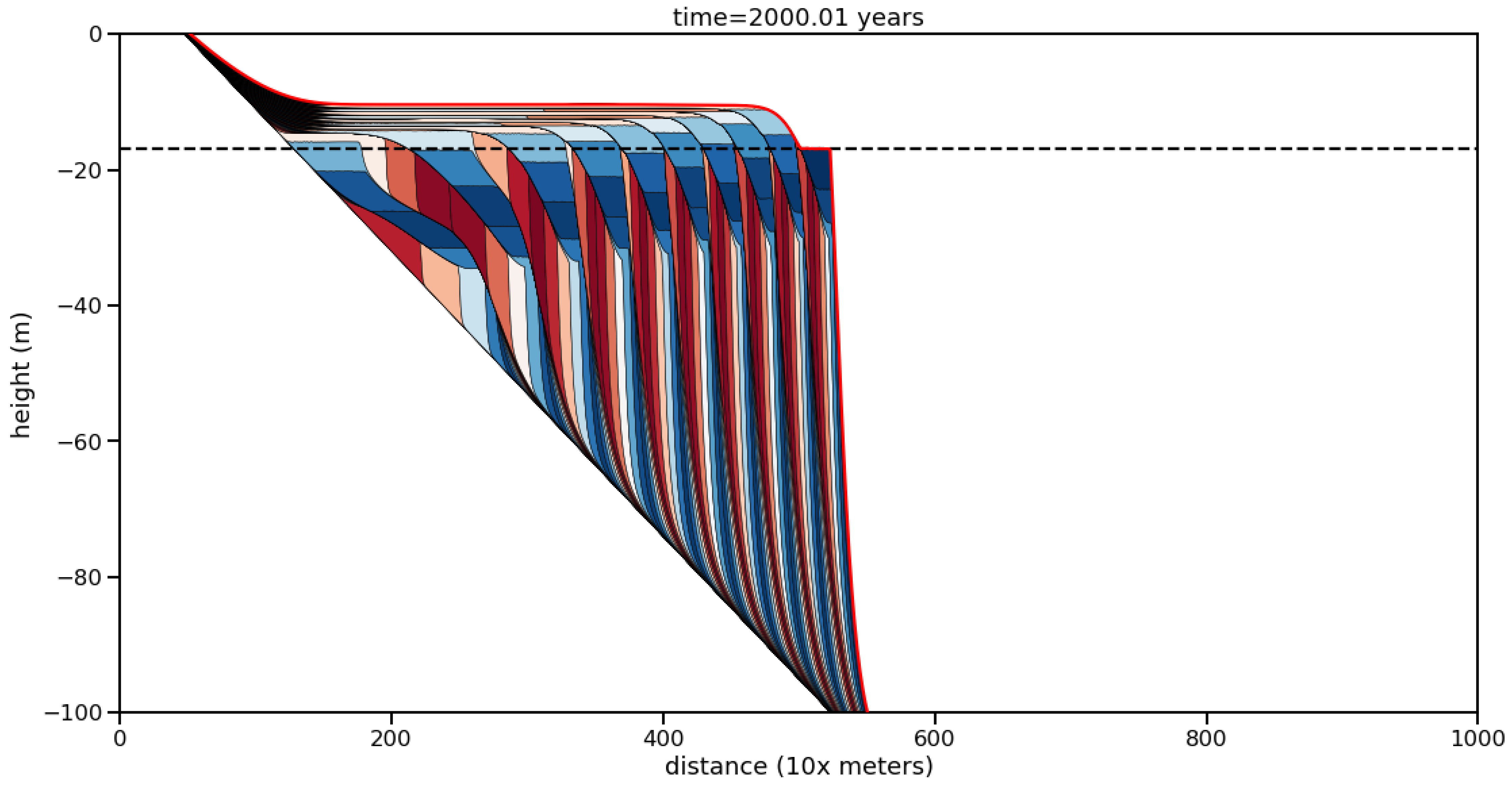
Over the next few weeks in your assignments you will be building a 1-D diffusive transport model to explore how variations in the following properties change the stratigraphic **architecture** of a basin:

1. transport
2. sediment supply
3. accommodation space

An example: varying accommodation space

In [10]:

```
animate_beds(beds=beds,otime=otime,rsl=rsl,aspect=5, ymin=-100)
```



For next Monday, please come prepared to discuss and work our way through the posted reading (Kenyon and Turcotte, 1985)