EOS 423 // EOS 518

Lab 1.2: Sedimentary Transport (Numerical Model)

Due: 1:30 pm January 31, 2025

You have one week to complete this assignment and upload your responses as a PDF to Brightspace. You are not excluded from working with others (pairs are recommended), but each person will submit their own copy of the assignment. **Responses to questions should be typed, using complete sentences and standard grammar.** Double check that your image resolution is high enough to read. If you write your response in a word processor, please export to .PDF before submitting your response.

A pair of students will be randomly selected on Monday to initiate and lead a discussion of the assignment. Be prepared to show your progress and discuss any challenges you still face.

Question	1	2	3	**Total:**
Marks:	5	5	2	12
Score:				

In this lab you will develop a transport model that is considerably more flexible than your previous approach. This new model will be used to explore how sediment flux and relative sea level impact transport and stratigraphy.

Bulk sediment transport: diffusion

The change of elevation over time is proportional to the second partial derivative of the topography with respect to space (the curvature). This equation is sometimes referred to as the *hillslope application* of the **diffusion equation**:

$$\frac{\partial h}{\partial t} = K \frac{\partial^2 h}{\partial x^2} \qquad \text{(diffusion equation)}$$

The diffusion equation is derived from combining the **continuity equation**, which expresses the conservation of mass:

$$\frac{\partial h}{\partial t} = -\frac{\partial S}{\partial x} \qquad \text{(continuity equation)}$$

with an expression of sediment transport rate, S, as a **diffusive flux**. In this formulation, S is linearly proportional to slope:

$$S = -K \frac{\partial h}{\partial x} \qquad \text{(diffusive flux)}$$

In part **1.1** of this assignment we looked at a specific application of bulk (diffusive) transport with an *analytical* solution. This you will be designing a numerical model of bulk transport using an *implicit* finite difference scheme (specifically the Crank-Nicolson algorithm). The questions below are designed to showcase that your model is correctly solving the diffusion equation.

Crank-Nicholson implicit scheme

The Crank-Nicolson method is numerically stable, implicit, and second-order in time – $O(\Delta t^2)$. The method solves for the next time-step iteration of the system by taking the average of the central-difference estimate of the second partial derivative of the topography with respect to space at both the current time step and the future timestep. Red terms are unknown at the current timestep (this is the *implicit* part of the numerical method).

$$\frac{h_i^{t+\Delta t} - h_i^t}{\Delta t} = \frac{K}{2}\left(\frac{h_{i-\Delta x}^t - 2\ h_i^t + h_{i+\Delta x}^t}{\Delta x^2} + \frac{h_{i-\Delta x}^{t+\Delta t} - 2\ h_i^{t+\Delta t} + h_{i+\Delta x}^{t+\Delta t}}{\Delta x^2}\right)$$

It is useful to define r as (this number is sometimes called the Fourier number):

$$r = K \frac{\Delta t}{2\ \Delta x^2} \qquad \text{(Fourier number)}$$

to simplify the equation to the following form:

$$h_i^{t+\Delta t} - h_i^t = r\ (h_{i-\Delta x}^t - 2\ h_i^t + h_{i+\Delta x}^t + h_{i-\Delta x}^{t+\Delta t} - 2\ h_i^{t+\Delta t} + h_{i+\Delta x}^{t+\Delta t})$$

This equation forms a system of linear equations that can be rearranged and solved using linear algebra. The general form of your problem (once you have collected unknowns on one side and knowns on the other) will be:

$$Ah^{t+\Delta t} = Bh^t + b^t$$

where A and B are square matrices with whose side-length is equal to the length of h and b is a vector of boundary conditions (additional flux in or out of each finite element in h_i). The matrix equation above is solved by multiplying both sides by A^{-1}:

$$h^{t+\Delta t} = A^{-1}(Bh^t + b^t)$$

Questions

Make a model with the following initial boundary conditions. Your topographic profile should cover 10 kilometers. The initial topography will be randomly generated as a cumulative sum of 1000 random draws from a normal distribution with $\mu = 0$ and $\sigma = 1$. Your left boundary condition will be +10 m, and your right boundary condition will be −20 m. The diffusivity, K, should be set to 2×10^2 (m^2/yr).

```
import numpy as np

#set your random number seed to 2:
np.random.seed(2)

#initial topography (1000 length covers 10 km with dx=10):
H=np.cumsum(np.random.normal(0,1,1000))+10

#fix right and left boundaries
left_boundary = 10
right_boundary = -20

#set diffusivity
K = 2e2
```

(if you are not using python, initiate your initial topography with a similar approach)

Question 1 (5)
Make a plot showing topography (y-axis) versus distance (x-axis) for your model at the start, after 500 years, and after 50,000 years.

Question 2 (5)
Starting with the equilibrium topography (the 50,000 year surface) from question 1, introduce a constant sediment flux of 10 m^2 per year at the coastline (where topography first drops below 0). Make a plot showing the new topography (y-axis) versus distance (x-axis) for your model after 50 years, and after 1,000 years.

Question 3 (2)

Show that the amount of sediment added (your flux multiplied by time) is equal to the total change in topography.