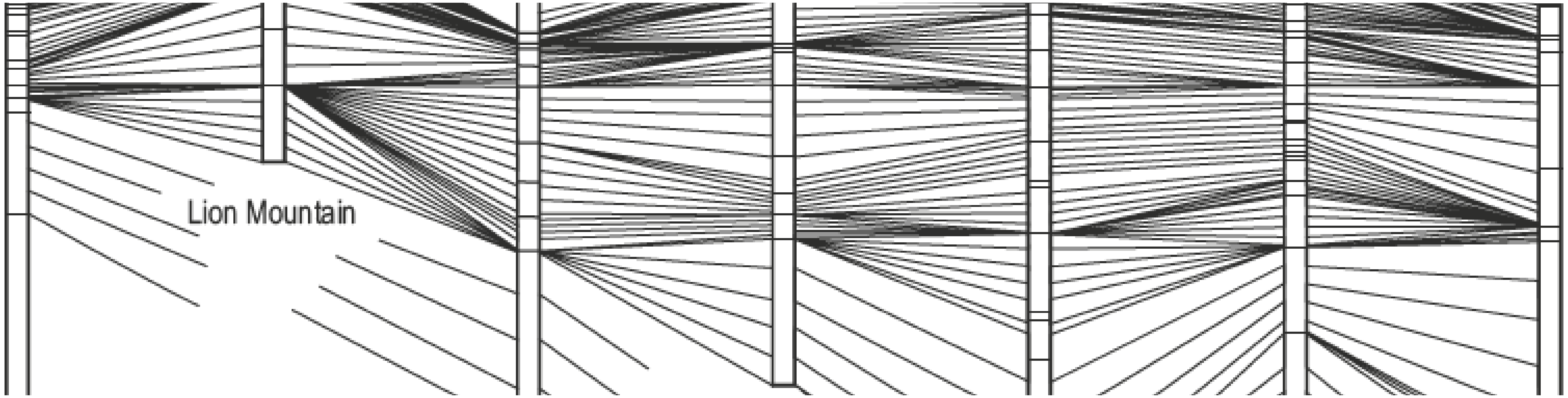


Lecture 13: Age models part 2

1. Building an age model
    - dealing with uncertainty
2. Introduction to carbonates

We acknowledge and respect the lək̓ʷəŋən peoples on whose traditional territory the university stands and the Songhees, Esquimalt and W̱SÁNEĆ peoples whose historical relationships with the land continue to this day.

In [58]:

```
from matplotlib.patches import Polygon
import scipy.stats as stats
from scipy.interpolate import interp1d
from matplotlib import pyplot as plt
import numpy as np
```

In [59]:

```python
def rando_strat(ax,num_box,height,boxes,liths):
    box_colors= [('#FFB142',0.6),('#33D9B2',0.5),('#34ACE0',0.4),('#706FD3',0.3),('#2C2C54',0.2),('#84817A',0.1)]
    if len(boxes)==0:
        boxes=np.cumsum(np.random.gamma(1,1, num_box))
        boxes=boxes/boxes[-1]*height
        boxes=np.hstack((boxes[0],np.diff(boxes)))
        liths=np.random.randint(0,len(box_colors),num_box)
    stack=0
    trace=[]
    for i,s in enumerate(boxes):
        tmp_w=box_colors[liths[i]][1]
        tmp_color=box_colors[liths[i]][0]
        #add a lithostratigraphy box
        xy=np.array([(0,stack),
                     (0+tmp_w,stack),
                     (0+tmp_w,stack+s),
                     (0,stack+s)])
        rect = Polygon(xy,closed=True,
                      facecolor=tmp_color,edgecolor="k",lw=0.5)
        ax.add_patch(rect)
        #grow the net record
        trace.append((stack,stack+s,tmp_w))
        stack=stack+s
    #set y-limits (from height of this column)
    ax.set_ylim([-0.1,height+0.01*height])
    #set x-limits (section-specific max width of strat box)
    ax.set_xlim([0,1])
    #turn off axes and frame
    ax.axis('off')
    return ax,np.array(trace),boxes,liths
```

In [60]:

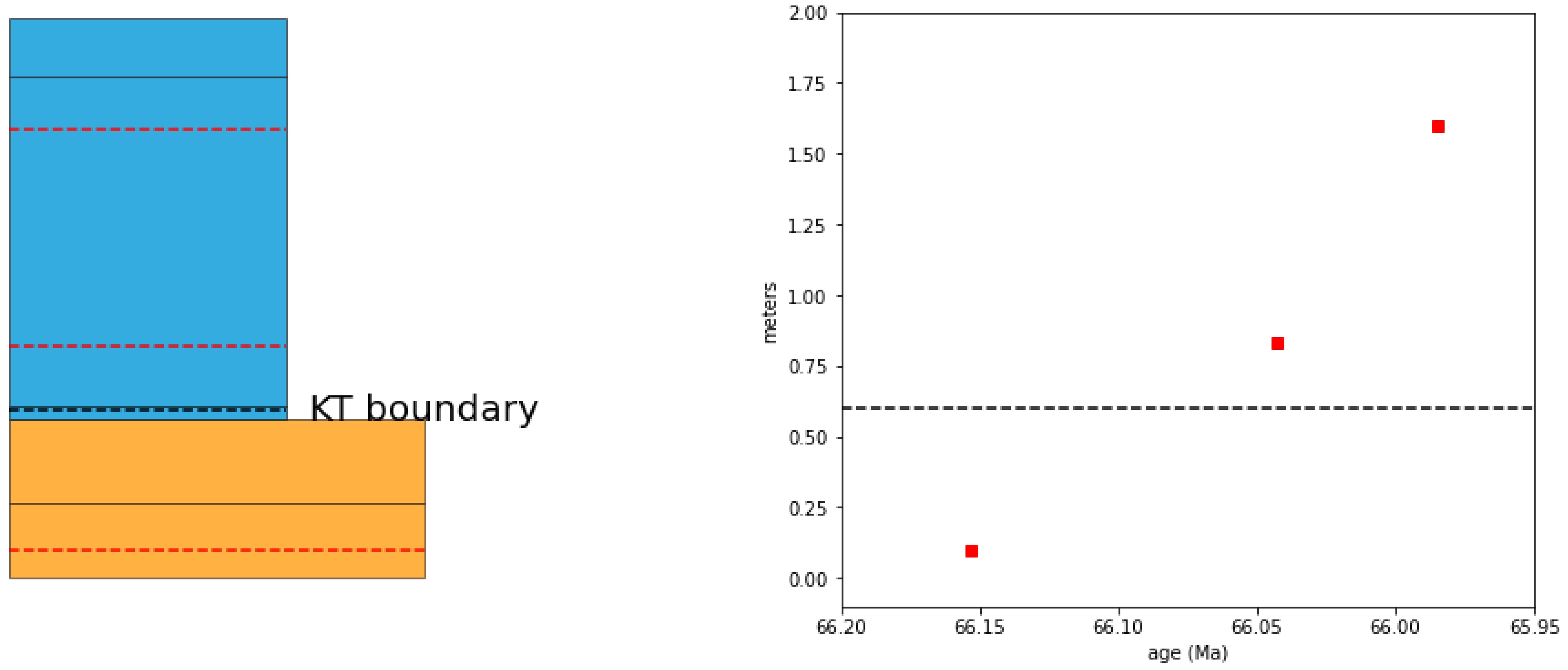
```python
def calc_age(ash_table,h):
    up=ash_table[(ash_table[:,1])>h][0]
    down=ash_table[(ash_table[:,1])<h][-1]
    sed_rate=(up[1]-down[1])/(down[0]-up[0]) #[1] is height, [0] is age

    h_diff=h-down[1]
    h_age=down[0]-h_diff/sed_rate

    return(h_age)

ashes={'ash1':{'height':0.1,
            'age': 66.153,
            '2sd':0.029},
       'ash2':{'height':0.83,
            'age': 66.043,
            '2sd':0.031},
       'ash3':{'height':1.6,
            'age': 65.985,
            '2sd':0.011}}
ash_table=np.array([(ashes[a]['age'],ashes[a]['height']) for a in ashes])
KT=0.6 #height in meters
```

In [61]:

```
def plot_ash(boxes = [], liths = []):
    fig=plt.figure(1,figsize=(15,6))
    ax=fig.add_subplot(121)
    ax,trace,boxes,liths=rando_strat(ax,num_box=5,height=2,boxes=boxes,liths=liths) #stratigraphy for fun
    #ashes in the strat column
    for a in ashes:
        tmp_w=trace[(trace[:,0]<=ashes[a]['height']) & (trace[:,1]>ashes[a]['height']),2]
        ax.plot([0,tmp_w],[ashes[a]['height'],ashes[a]['height']],'r--')
    #KT boundary
    tmp_w=trace[(trace[:,0]<=KT) & (trace[:,1]>KT),2]
    ax.plot([0,tmp_w],[KT,KT],'k--')
    ax.text(tmp_w,KT,'  KT boundary',verticalalignment='center',fontsize=20)
    #ash ages
    ax=fig.add_subplot(122)
    for a in ashes:
        ax.plot(ashes[a]['age'],ashes[a]['height'],'rs')
    ax.plot([66.2,65.95],[KT,KT],'k--')
    ax.set_xlim([66.2,65.95]); ax.set_ylim([-0.1,2])
    ax.set_ylabel('meters'); _=ax.set_xlabel('age (Ma)')
    return boxes, liths, fig
```

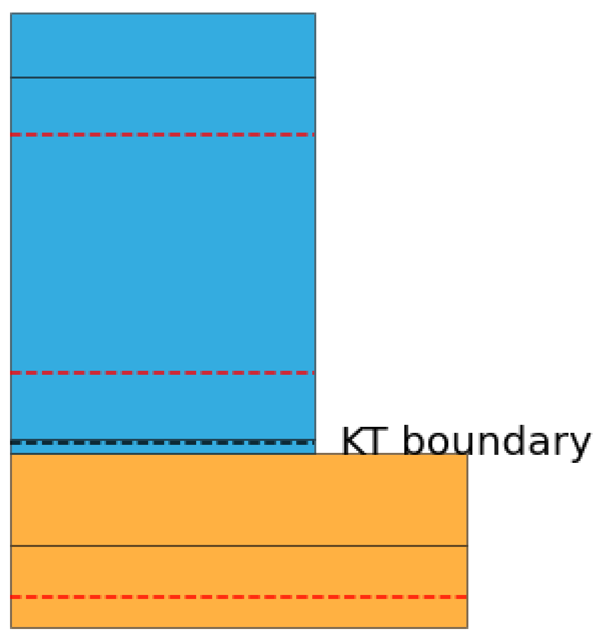
Building an age model

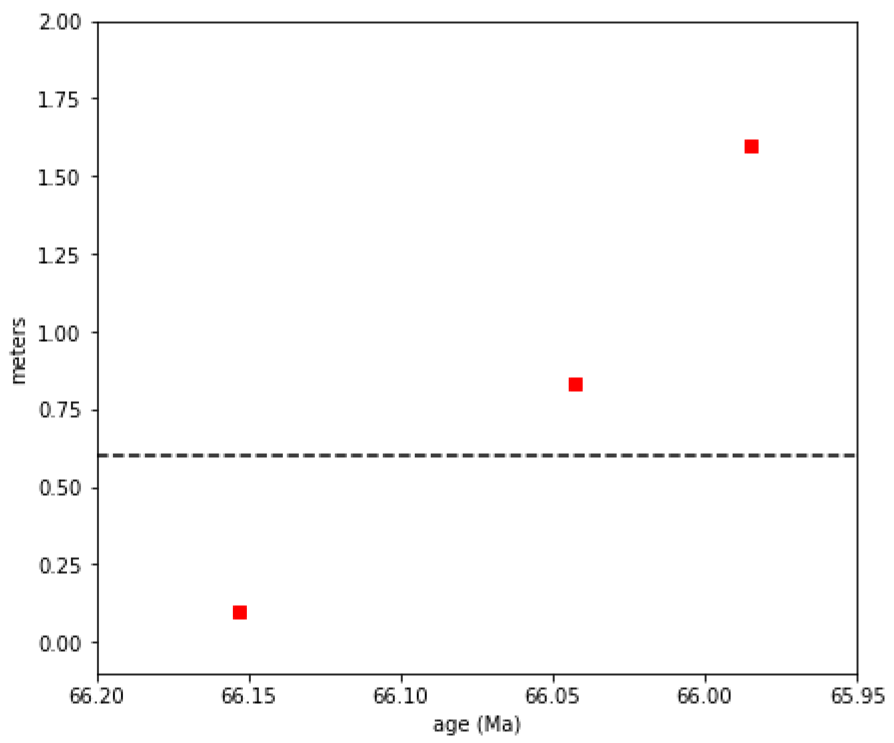
In [62]:
```
boxes, liths, _ = plot_ash()
```



Building an age model

In [62]:
```
boxes, liths, _ = plot_ash()
```





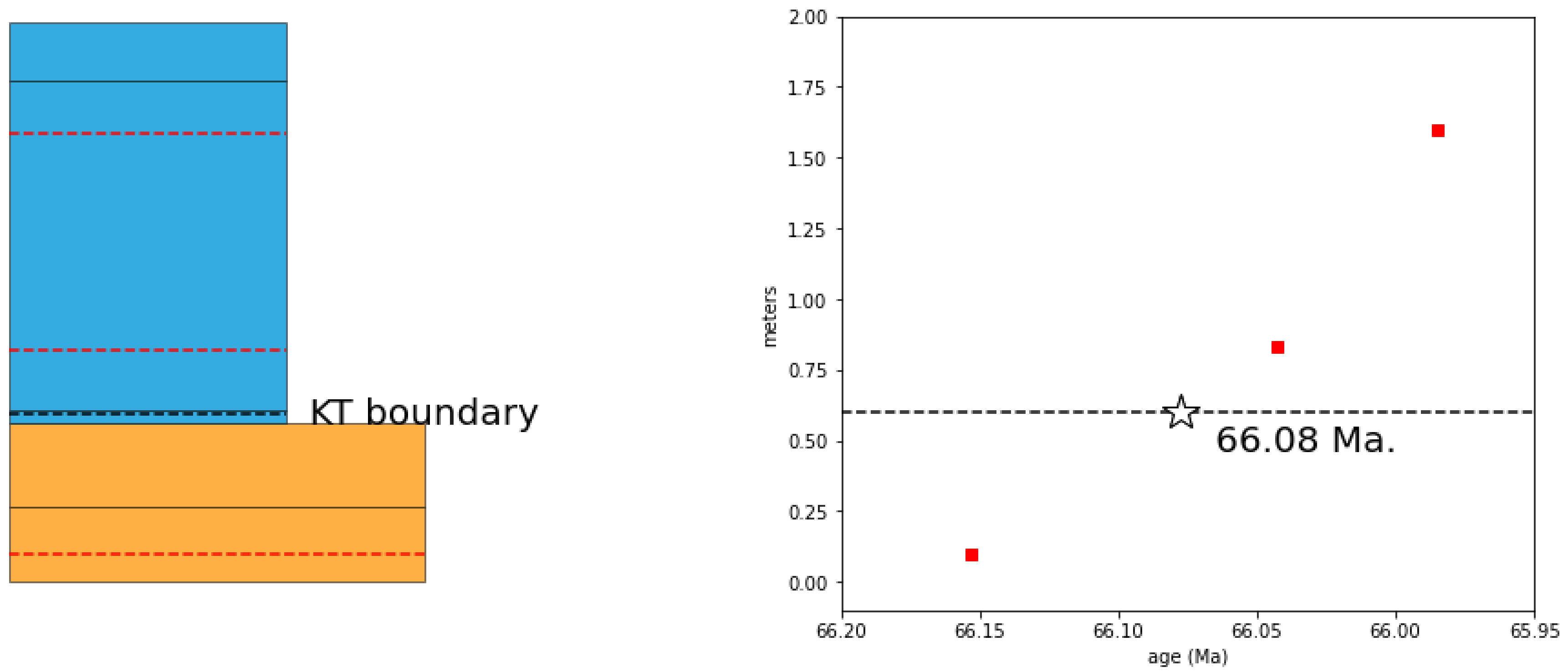
What is the age of the *KT Boundary*? What ages are possible? What ages are most likely?

Building an age model

Starting simple since we cant assume slow then fast is any more likely than fast then slow. Considering a constant sedimentation rate (a linear interpolation)..

In [63]:

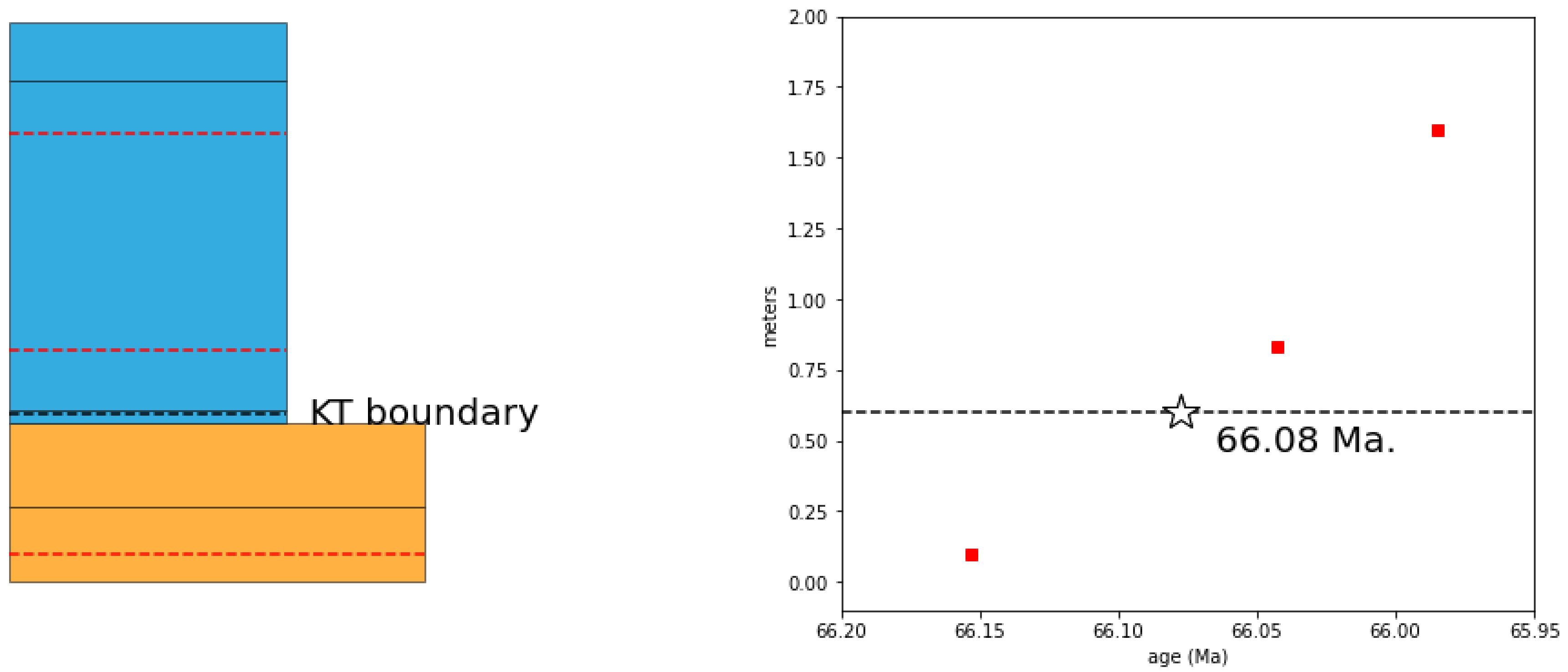
```
_,_, fig = plot_ash(boxes=boxes,liths=liths)
#calculate KT age
ax = fig.axes[-1]
KT_age=calc_age(ash_table,KT)
ax.plot([KT_age,KT_age],[KT,KT],'w*',mec='k',markersize=20)
_=ax.text(KT_age,KT-.1,'   %2.2f Ma.' % (KT_age),horizontalalignment='left',verticalalignment='center',fontsize=20)
```



Building an age model

Starting simple since we cant assume slow then fast is any more likely than fast then slow. Considering a constant sedimentation rate (a linear interpolation)..

In [63]:
```
_,_, fig = plot_ash(boxes=boxes,liths=liths)
#calculate KT age
ax = fig.axes[-1]
KT_age=calc_age(ash_table,KT)
ax.plot([KT_age,KT_age],[KT,KT],'w*',mec='k',markersize=20)
_=ax.text(KT_age,KT-.1,'   %2.2f Ma.' % (KT_age),horizontalalignment='left',verticalalignment='center',fontsize=20)
```

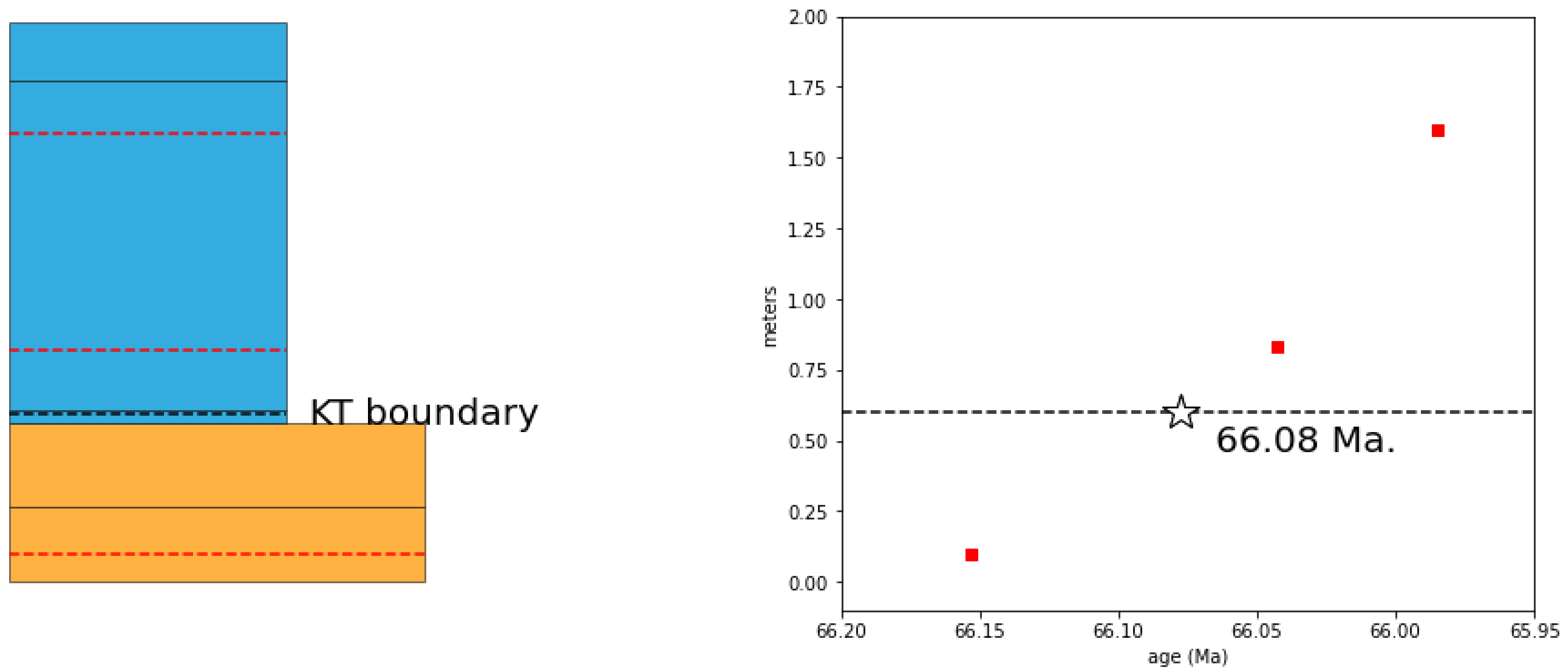


Even with the simple model.. what havn't we considered?

Building an age model

Starting simple since we cant assume slow then fast is any more likely than fast then slow. Considering a constant sedimentation rate (a linear interpolation)..

In [64]:
```
_,_, fig = plot_ash(boxes=boxes,liths=liths)
#calculate KT age
ax = fig.axes[-1]
KT_age=calc_age(ash_table,KT)
ax.plot([KT_age,KT_age],[KT,KT],'w*',mec='k',markersize=20)
_=ax.text(KT_age,KT-.1,'    %2.2f Ma.' % (KT_age),horizontalalignment='left',verticalalignment='center',fontsize=20)
```

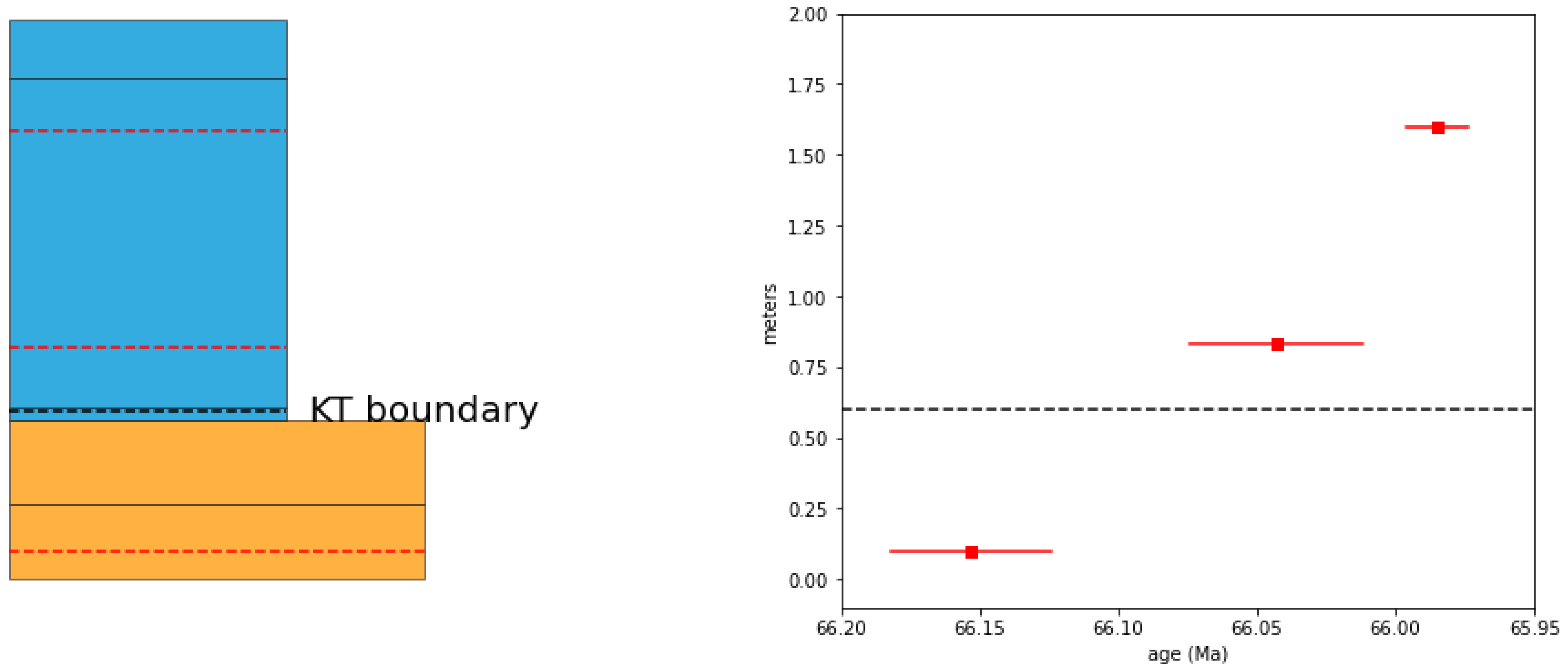


Even with the simple model.. what havn't we considered? Uncertainty in the geochronology

Building an age model

In [65]:

```
_,_, fig = plot_ash(boxes=boxes,liths=liths)
ax = fig.axes[-1]
for a in ashes:
    ax.plot(ashes[a]['age'],ashes[a]['height'],'rs')
    ax.plot([ashes[a]['age']+ashes[a]['2sd'],ashes[a]['age']-ashes[a]['2sd']],
            [ashes[a]['height'],ashes[a]['height']],'r-')
```



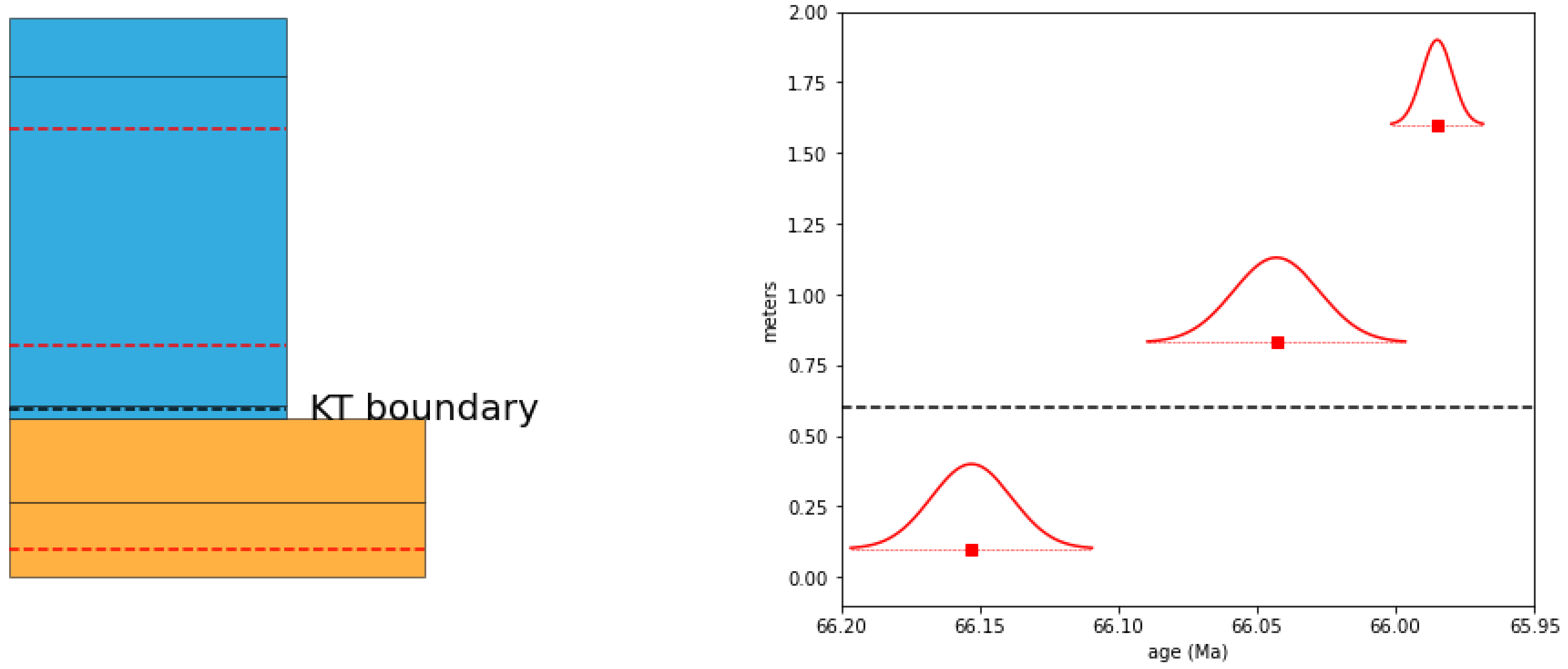
Building an age model

How can we describe the uncertainty of the ash ages?

Building an age model

In [66]:

```
_,_, fig = plot_ash(boxes=boxes,liths=liths)
ax = fig.axes[-1]
for a in ashes:
    mu=ashes[a]['age'] #normal distribution
    sigma=ashes[a]['2sd']/2
    x = np.linspace(mu - 3*sigma, mu + 3*sigma, 300)
    distro=stats.norm.pdf(x, mu, sigma) #scaled for display
    distro=distro/max(distro)*0.3
    ax.plot(x, distro+ashes[a]['height'],'r')
    #ash strat height
    ax.plot([ashes[a]['age']+ashes[a]['2sd']*3/2,ashes[a]['age']-ashes[a]['2sd']*3/2],
            [ashes[a]['height'],ashes[a]['height']],'r--',lw=0.5)
```



Building an age model

How do we include these *normally distributed* uncertainties into our constant sedimentation rate age model?

Building an age model

How do we include these *normally distributed* uncertainties into our constant sedimentation rate age model?

Consider..

$$Z = (Y \pm \sigma_Y) + (X \pm \sigma_X)$$

What is Z? What is σ_Z?

Building an age model

How do we include these *normally distributed* uncertainties into our constant sedimentation rate age model?

Consider..

$$Z = (Y \pm \sigma_Y) + (X \pm \sigma_X)$$

What is Z? (sum of the means) What is σ_Z? (square root of $\Sigma\sigma^2$)

In [67]:

```
import numpy as np
Y = np.random.normal(2,.3,1000000)
X = np.random.normal(20,.2,1000000)
Z = Y + X
print(f"""Mean of Z: {np.mean(Z):.1f} and standard deviation of Z: {np.std(Z):.2f}
Analytical stdev: {(.3**2+.2**2)**(1/2):.2f}""")
```

```
Mean of Z: 22.0 and standard deviation of Z: 0.36
Analytical stdev: 0.36
```

Building an age model

How do we include these *normally distributed* uncertainties into our constant sedimentation rate age model?

Building an age model

How do we include these *normally distributed* uncertainties into our constant sedimentation rate age model?

Consider..

$$Z = (Y \pm \sigma_Y) \times (X \pm \sigma_X)$$

What is Z? What is σ_Z?

Building an age model

How do we include these *normally distributed* uncertainties into our constant sedimentation rate age model?

Consider..

$$Z = (Y \pm \sigma_Y) \times (X \pm \sigma_X)$$

What is Z? (product of the means) What is σ_Z? (square root of the sum of the relative errors squared times mean of Z)

$$\frac{\sigma_Z}{Z} \approx \left(\left(\frac{\sigma_Y}{Y} \right)^2 + \left(\frac{\sigma_X}{X} \right)^2 \right)^{\frac{1}{2}}$$

In [68]:

```
import numpy as np
Y = np.random.normal(2,.3,1000000)
X = np.random.normal(20,.2,1000000)
Z = Y * X
print(f"""Mean of Z: {np.mean(Z):.1f} and standard deviation of Z: {np.std(Z):.1f}
Analytical stdev: {np.mean(Z)*((0.3/2)**2+(0.2/20)**2)**(1/2):.2f}""")
```

```
Mean of Z: 40.0 and standard deviation of Z: 6.0
Analytical stdev: 6.01
```

Building an age model

Let's apply these rules to linear interpolation:

$$\frac{A_{KT} - A_0}{H_{KT} - H_0} = \frac{A_1 - A_0}{H_1 - H_0}$$

Building an age model

Let's apply these rules to linear interpolation:

$$\frac{A_{KT} - A_0}{H_{KT} - H_0} = \frac{A_1 - A_0}{H_1 - H_0}$$

$$A_{KT} = \left(\frac{A_1 - A_0}{H_1 - H_0}\right)(H_{KT} - H_0) + A_0$$

Building an age model

Let's apply these rules to linear interpolation:

$$\frac{A_{KT} - A_0}{H_{KT} - H_0} = \frac{A_1 - A_0}{H_1 - H_0}$$

$$A_{KT} = \left(\frac{A_1 - A_0}{H_1 - H_0}\right)(H_{KT} - H_0) + A_0$$

$$A_{KT} = \left(\frac{\color{red}{A_1 - A_0}}{H_1 - H_0}\right)(H_{KT} - H_0) + \color{red}{A_0}$$

Building an age model

Let's apply these rules to linear interpolation:

$$\frac{A_{KT} - A_0}{H_{KT} - H_0} = \frac{A_1 - A_0}{H_1 - H_0}$$

$$A_{KT} = \left(\frac{A_1 - A_0}{H_1 - H_0}\right)(H_{KT} - H_0) + A_0$$

$$A_{KT} = \left(\frac{\color{red}{A_1 - A_0}}{H_1 - H_0}\right)(H_{KT} - H_0) + \color{red}{A_0}$$

$$\sigma A_{kt}^2 = \left((\sigma A_0^2 + \sigma A_1^2)^{\frac{1}{2}} \times \frac{H_{KT} - H_0}{H_1 - H_0}\right)^2 + \sigma A_0^2$$

In [69]:

```
age = (
    lambda ashes, ktH: (ashes["ash2"]["age"] - ashes["ash1"]["age"])
    / (ashes["ash2"]["height"] - ashes["ash1"]["height"])
    * (ktH - ashes["ash1"]["height"])
    + ashes["ash1"]["age"]
)
```

In [70]:

```python
draws = []
for i in range(10000):
    simulated_ash = {
        "ash1": {
            "height": 0.1,
            "age": np.random.normal(66.153, 0.029 / 2),
            "2sd": 0.029,
        },
        "ash2": {
            "height": 0.83,
            "age": np.random.normal(66.043, 0.031 / 2),
            "2sd": 0.031,
        },
        "ash3": {
            "height": np.random.normal(66.153, 0.011 / 2),
            "age": 65.985,
            "2sd": 0.011,
        },
    }
    draws.append(age(simulated_ash, KT))
print(np.std(draws))
```

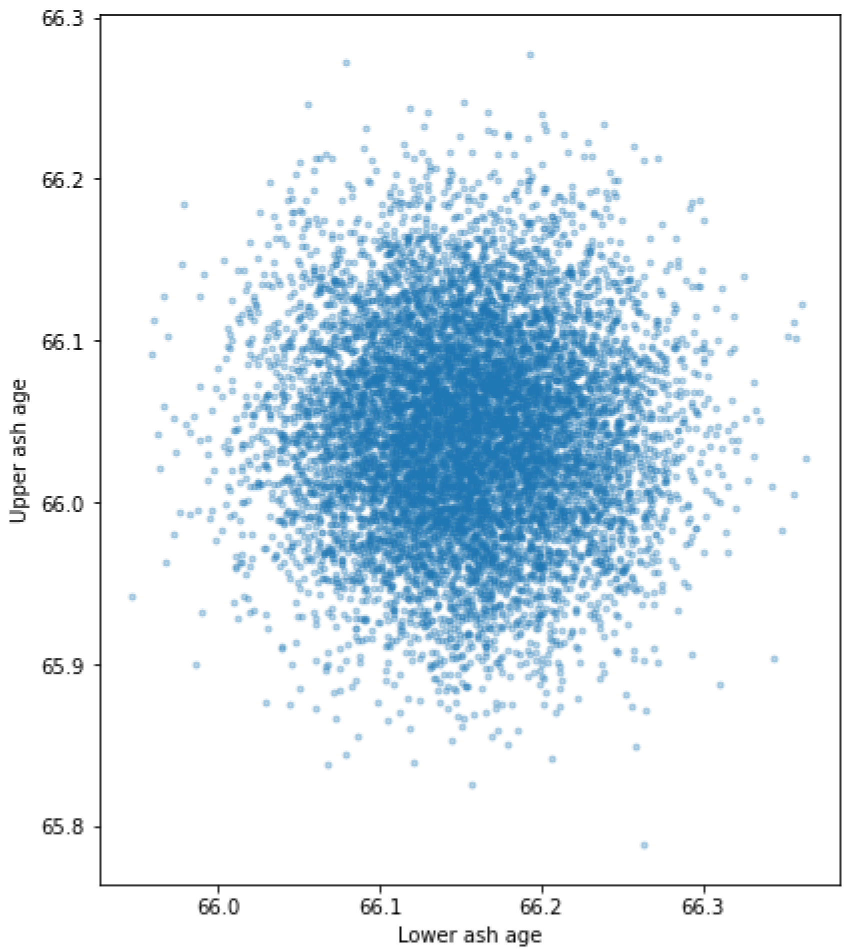
```
0.011619603147695575
```

In [71]:

```
plt.figure(figsize=(8,8))
a1 = np.random.normal(66.153, 0.029 * 2, 10000)
a2 = np.random.normal(66.043, 0.031 * 2, 10000)
plt.plot(a1,a2,'.',alpha=.3)
plt.gca().set_aspect('equal')
plt.gca().set_xlabel('Lower ash age')
plt.gca().set_ylabel('Upper ash age')
```

Out[71]:

```
Text(0, 0.5, 'Upper ash age')
```

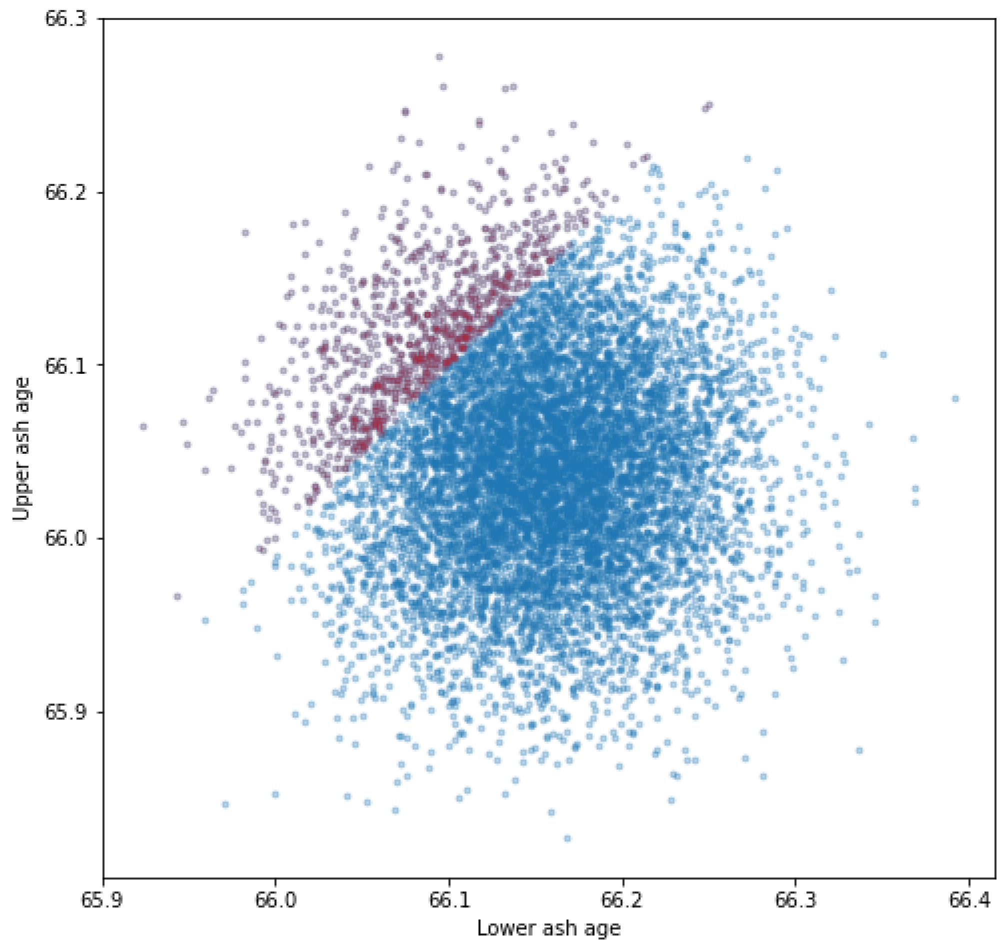


In [72]:

```
plt.figure(figsize=(8, 8))
a1 = np.random.normal(66.153, 0.029 * 2, 10000)
a2 = np.random.normal(66.043, 0.031 * 2, 10000)
plt.plot(a1, a2, ".", alpha=0.3)
plt.plot(a1[a1 < a2], a2[a1 < a2], "r.", alpha=0.1)
plt.gca().set_aspect("equal")
plt.gca().set_xlabel("Lower ash age")
plt.gca().set_ylabel("Upper ash age")
```

Out[72]:

```
Text(0, 0.5, 'Upper ash age')
```

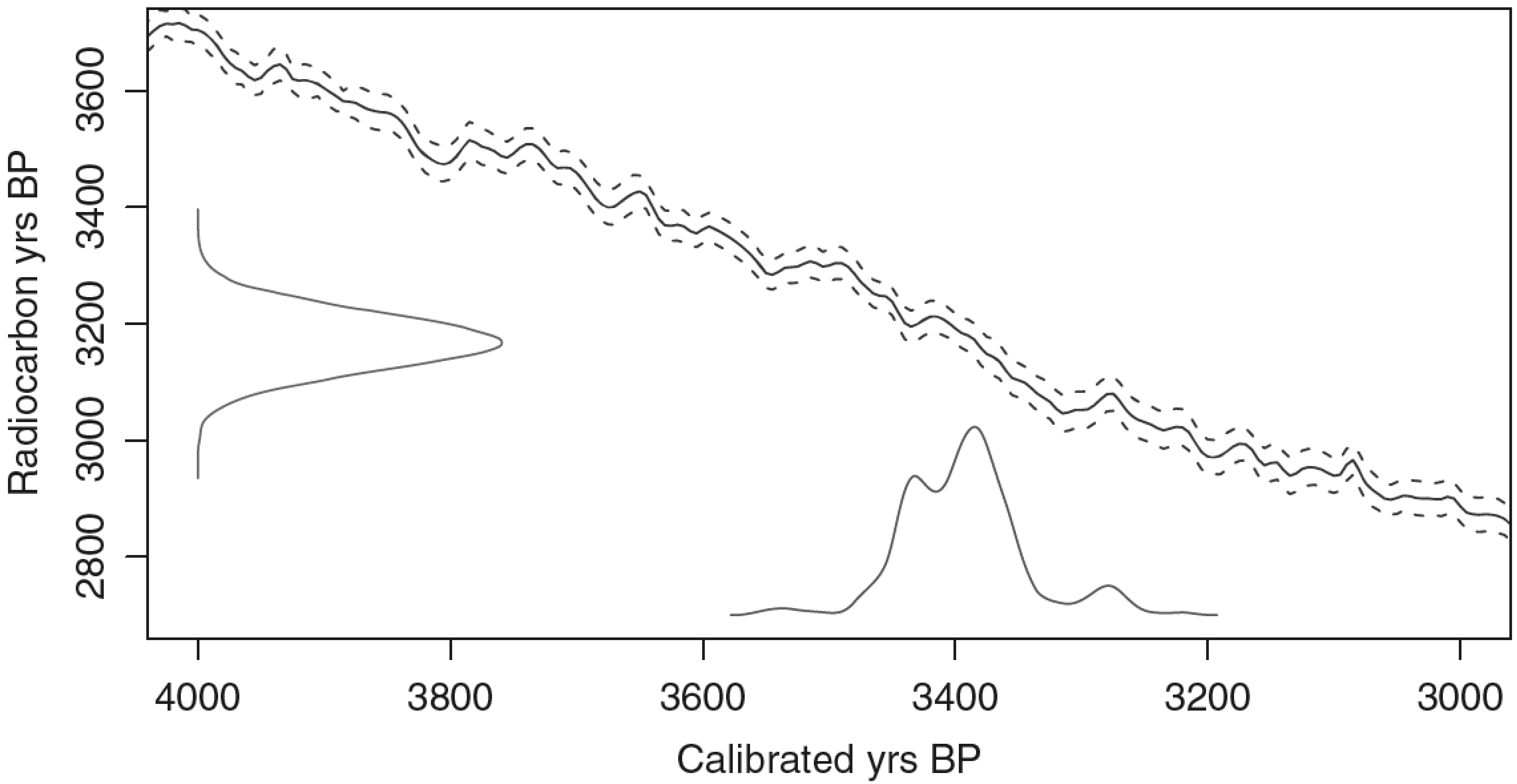


Building an age model

How do we include these *normally distributed* uncertainties into our constant sedimentation rate age model?

- Even in our simple model the uncertainty is not exactly normal (there is covariation)
- Random number generators allow us account to account for these more complicated forms of uncertainty (Monte Carlo methods)

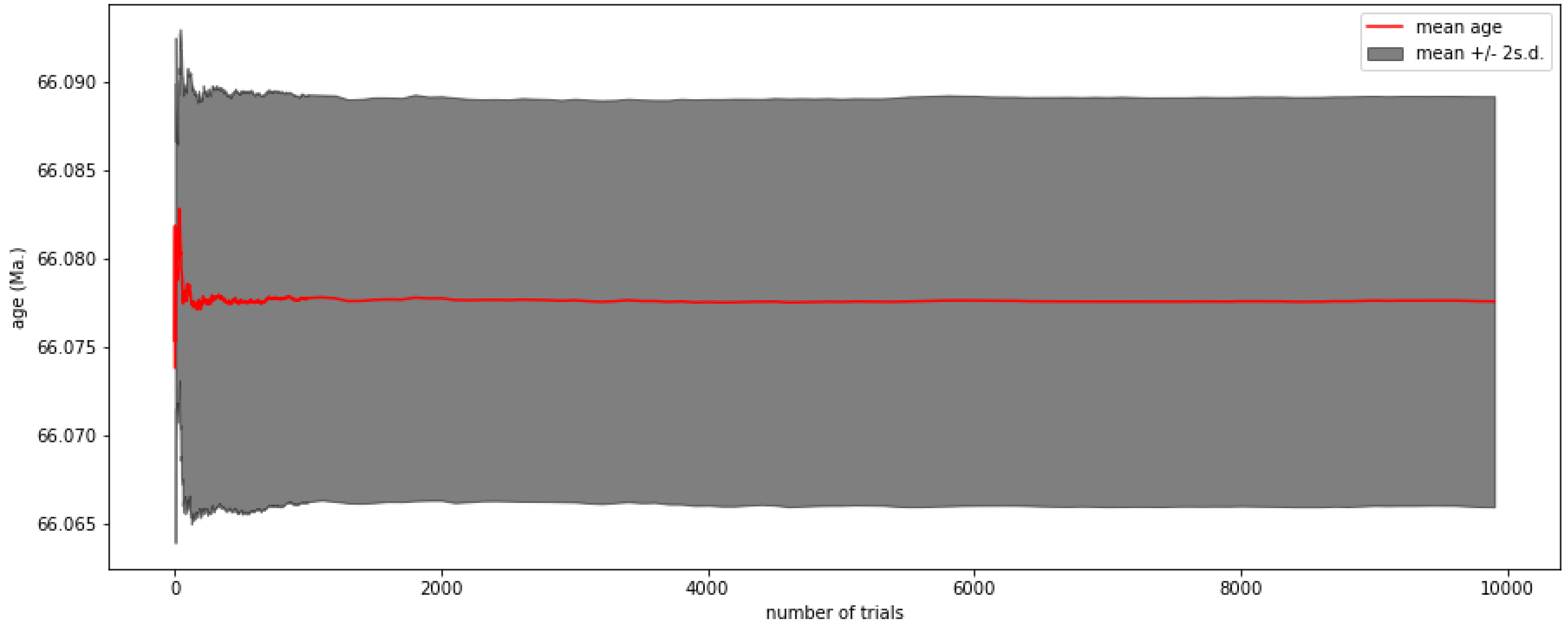
Power of Monte Carlo approaches



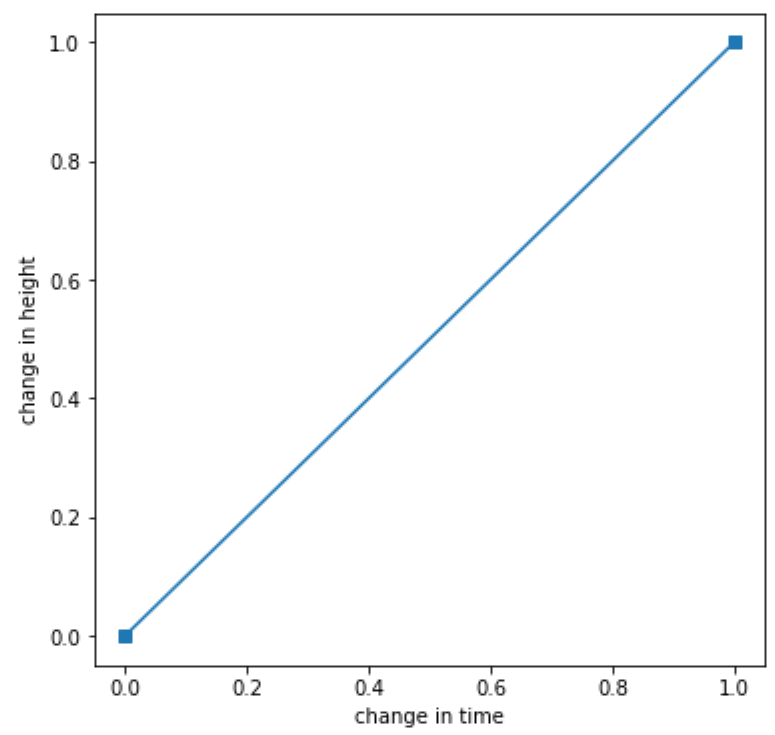
How many samples is enough?

In [77]:

```
fig=plt.figure(1,figsize=(15,6))
ax=fig.add_subplot(111)
idx=list(range(1000)) +list(range(1000,len(KT_ages)+1,100))
#statistics of KT age as function of number of trials
KT_evolve=[]
for i in idx[:10000]: #more trials here
    KT_evolve.append((np.mean(KT_ages[0:i+1]),np.std(KT_ages[0:i+1]),len(KT_ages[0:i+1])))
KT_evolve=np.array(KT_evolve)
#plot the results
ax.plot(KT_evolve[:,2],KT_evolve[:,0],'r-',label='mean age')
ax.fill_between(KT_evolve[:,2],KT_evolve[:,0]+KT_evolve[:,1],KT_evolve[:,0]-KT_evolve[:,1],
                color='k',alpha=0.5,zorder=0,label='mean +/- 2s.d.')
ax.set_ylabel('age (Ma.)'); ax.set_xlabel('number of trials');_=ax.legend(loc='best')
```



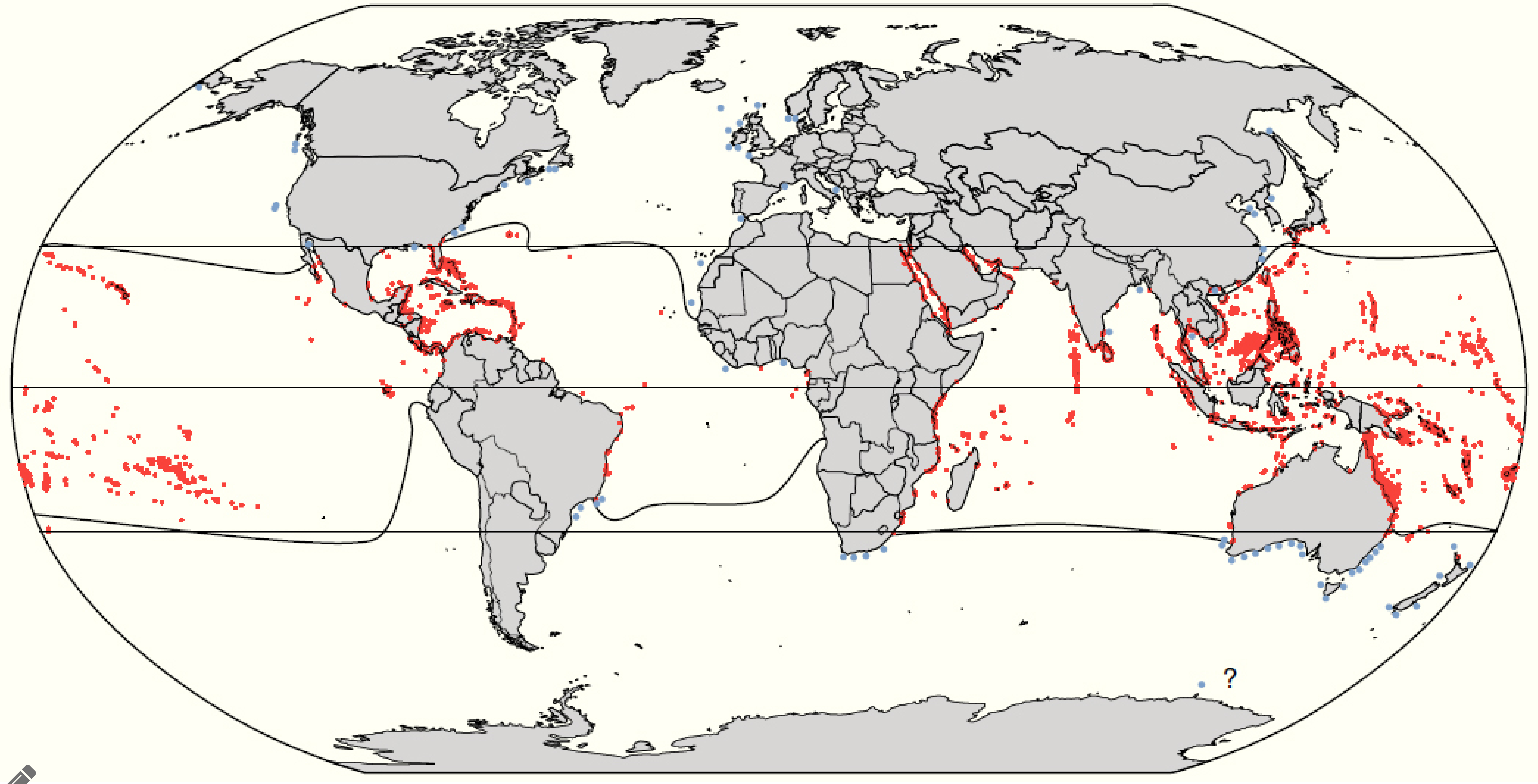
Constant sedimentation rate assumption

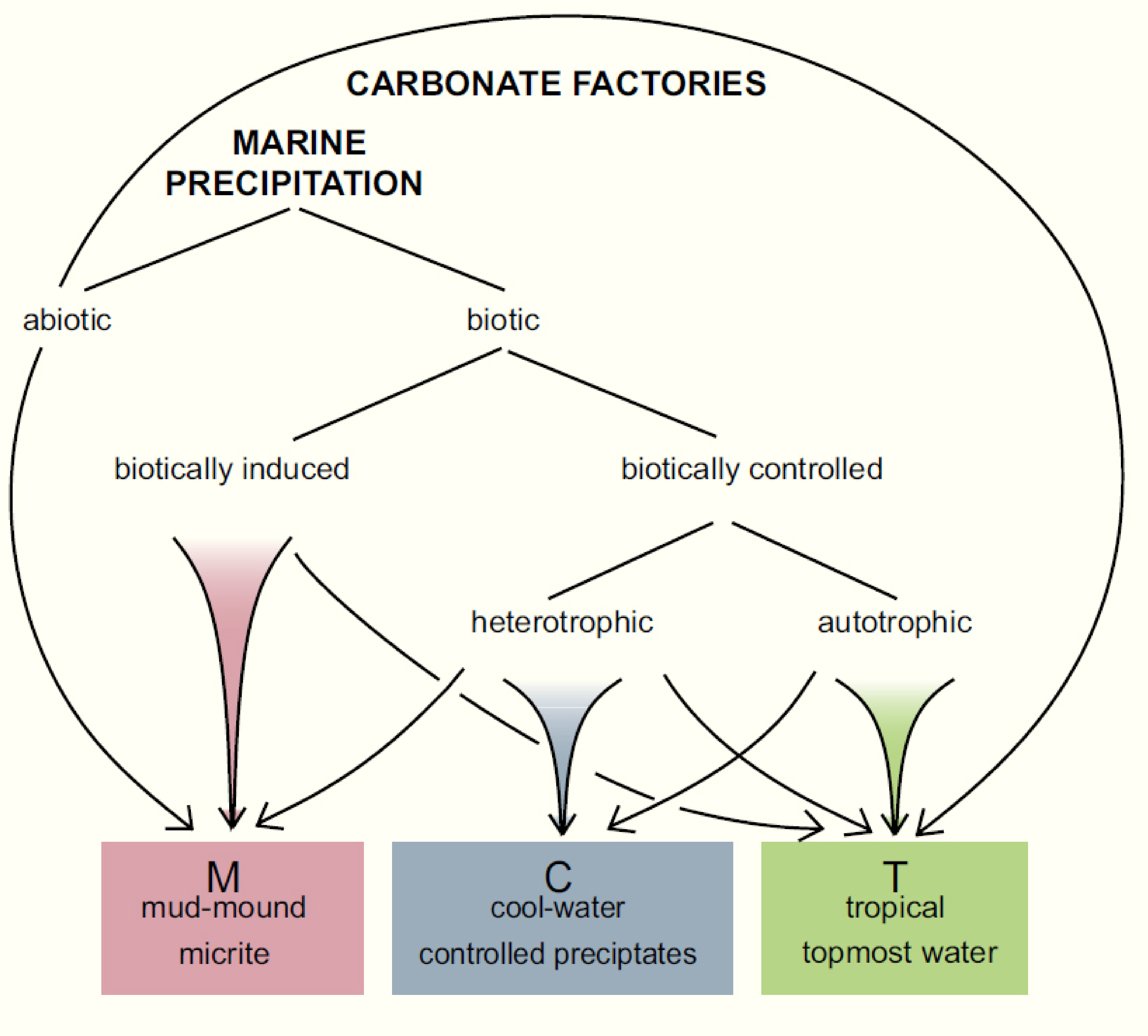


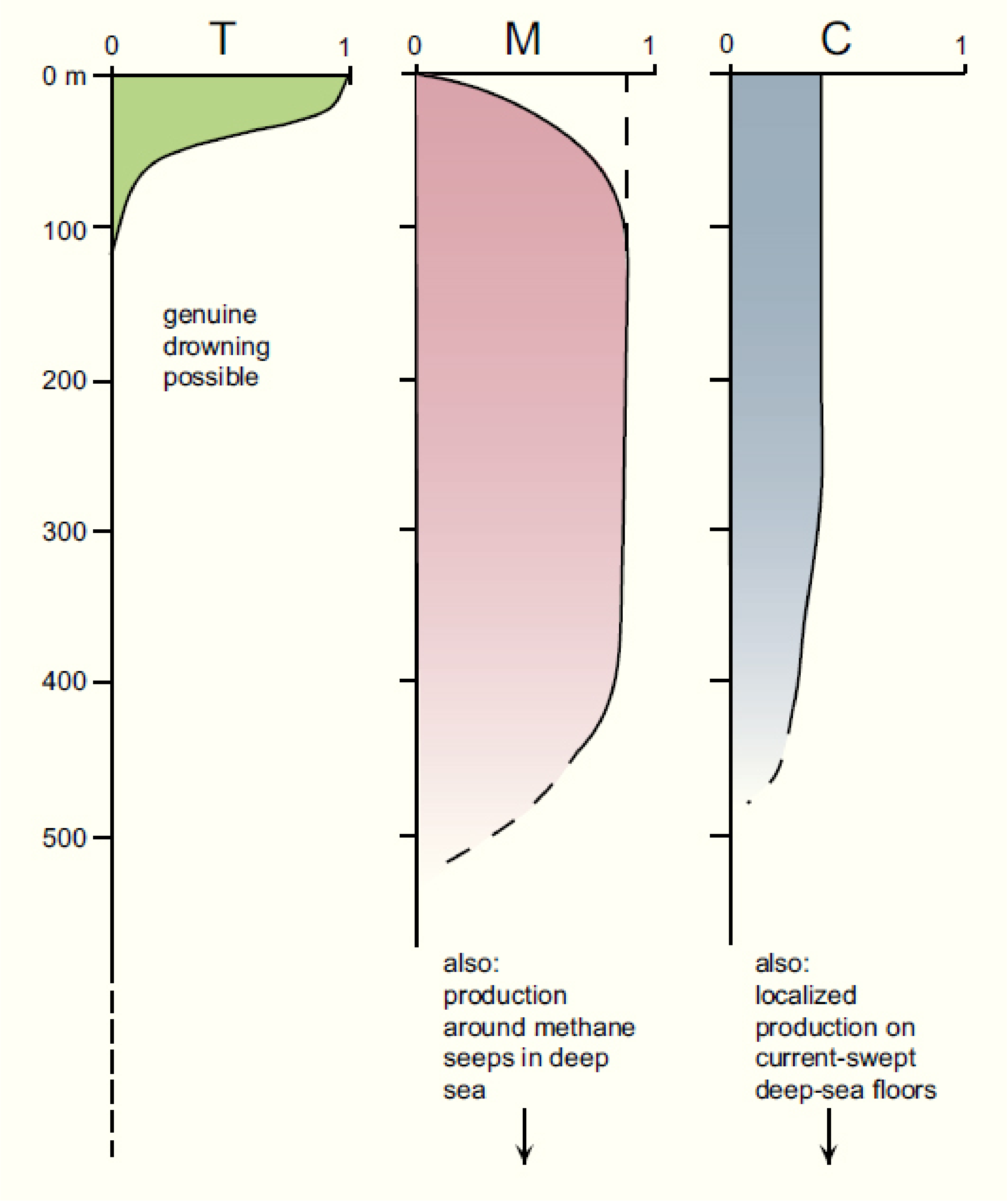
Constant sedimentation rate assumption

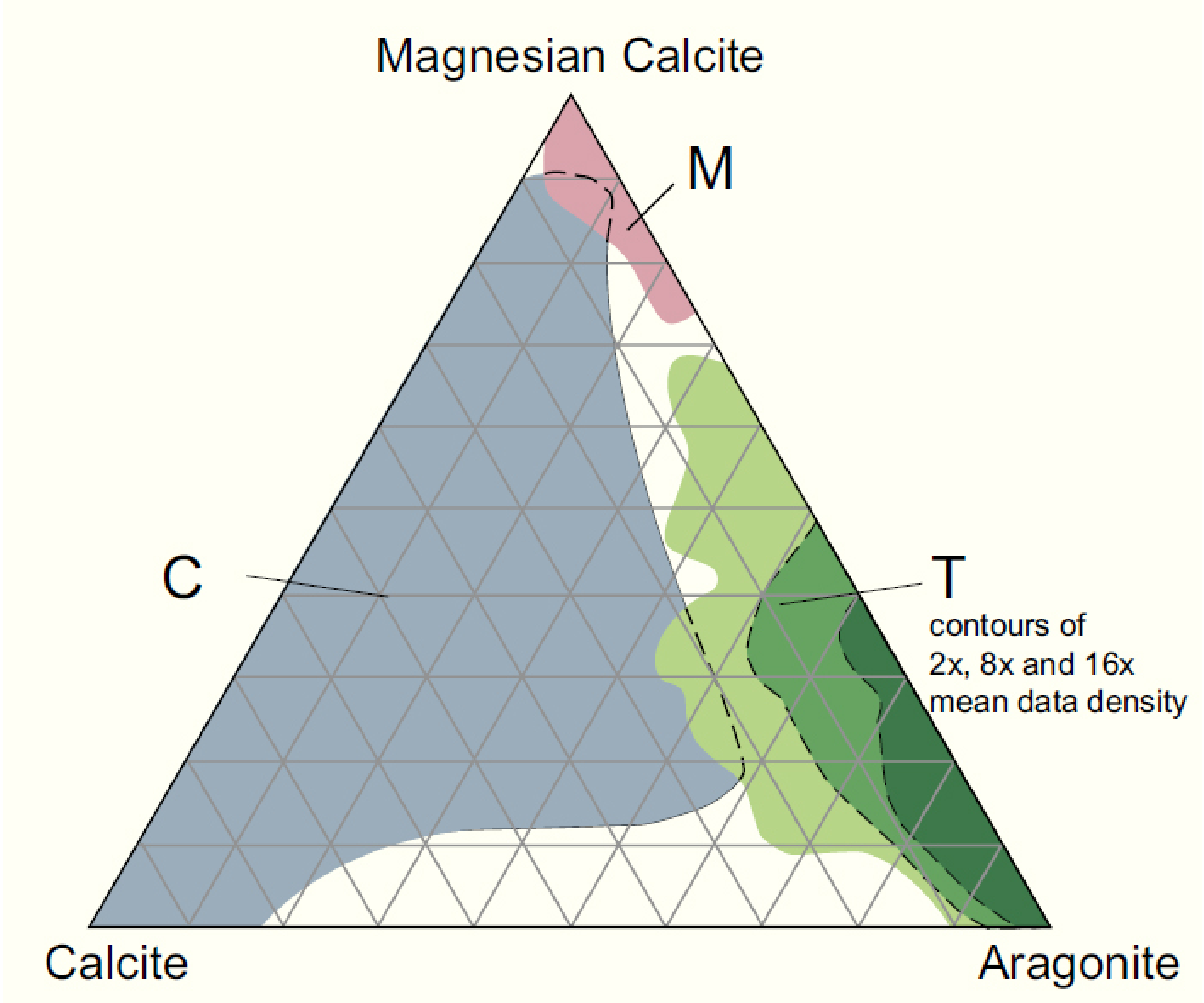
- goal in lab tomorrow: how can we get away from the assumption of constant sedimentation rate? what *should* it look like?

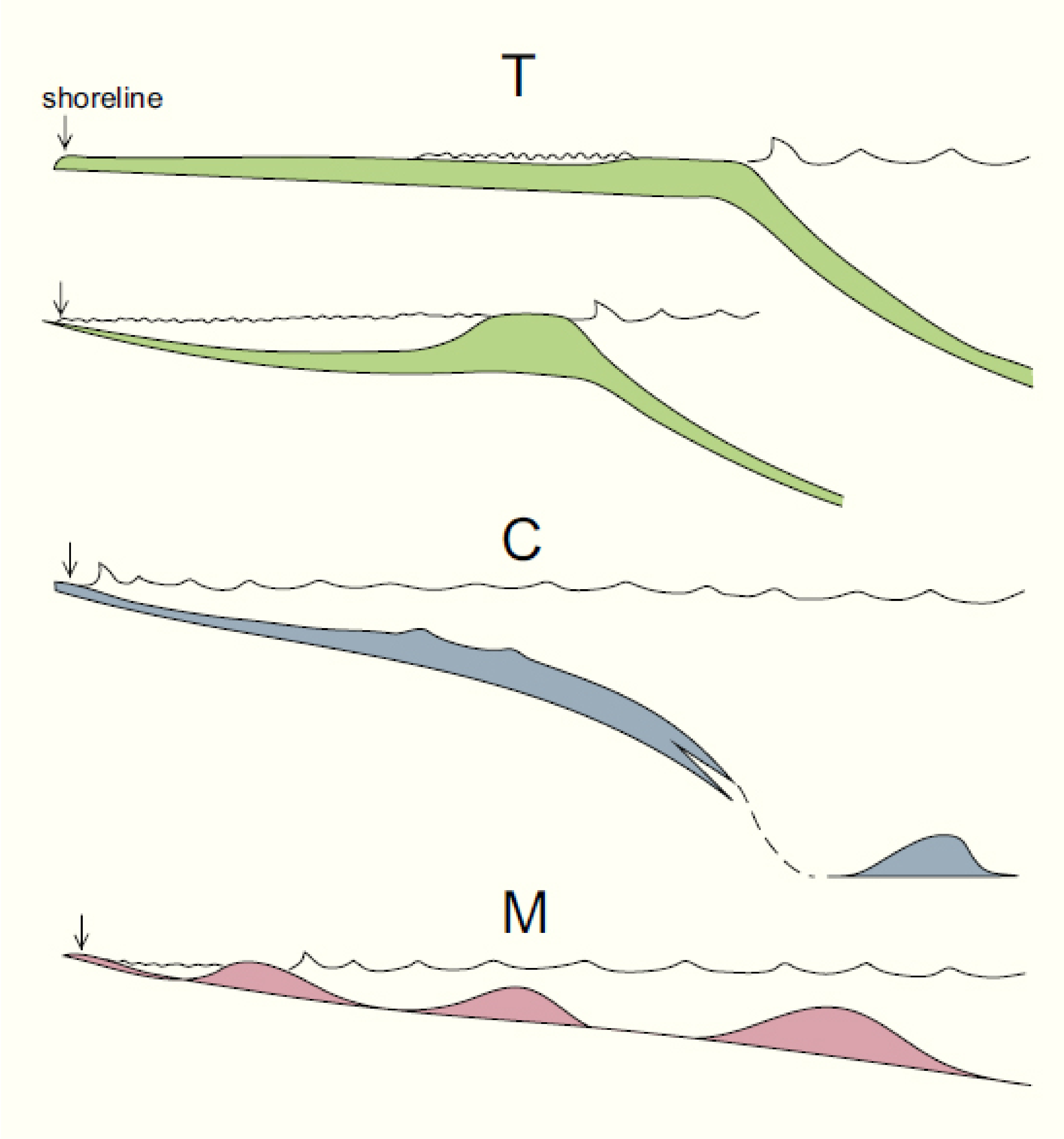
Carbonates




CARBONATE FACTORIES
MARINE PRECIPITATION
abiotic
biotic
biotically induced
biotically controlled
heterotrophic
autotrophic
M
mud-mound
micrite
C
cool-water
controlled preciptates
T
tropical
topmost water


T
M
C
0
1
0 m
100
200
300
400
500
genuine drowning possible
also: production around methane seeps in deep sea
also: localized production on current-swept deep-sea floors


Magnesian Calcite
M
C
T
contours of
2x, 8x and 16x
mean data density
Calcite
Aragonite


T
shoreline
C
M

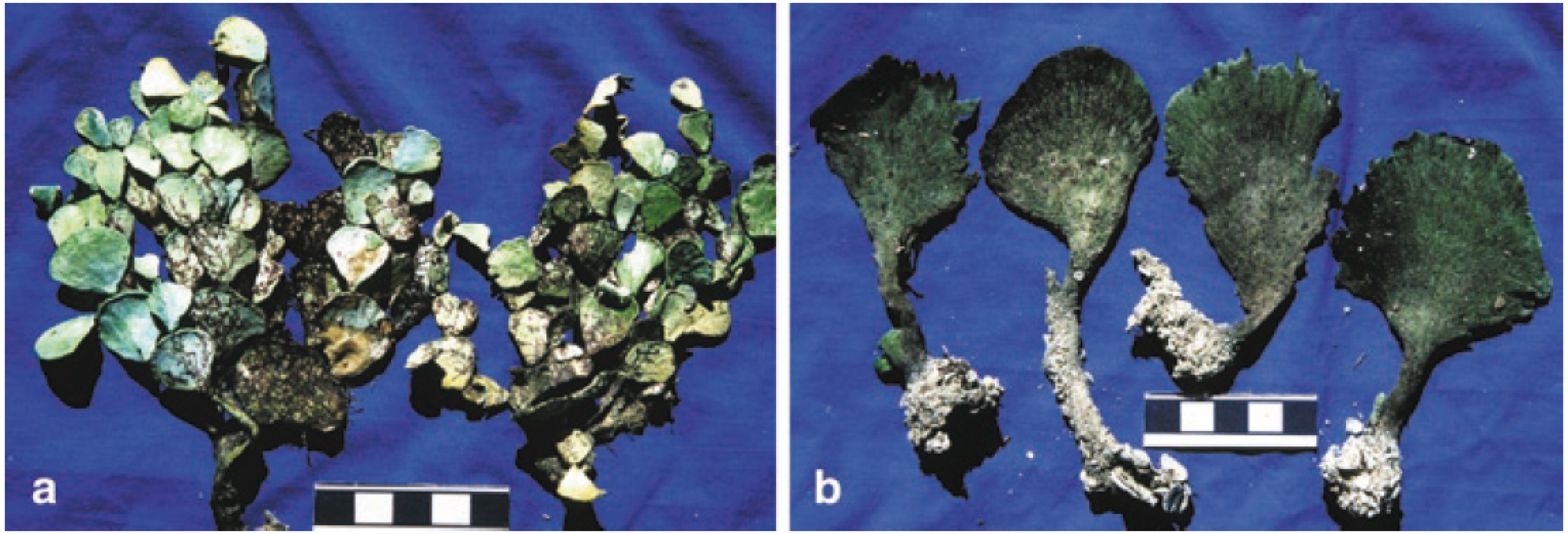


Fig. 4.11 Calcareous green algae that are non-calcified. **(a)** *Halimeda cuncata*, ~2 mwd. Recherche Archipelago, Albany Shelf, **(b)** *Rhipiliopsis peltata* ~2 mwd. Recherche Archipelago, Albany shelf

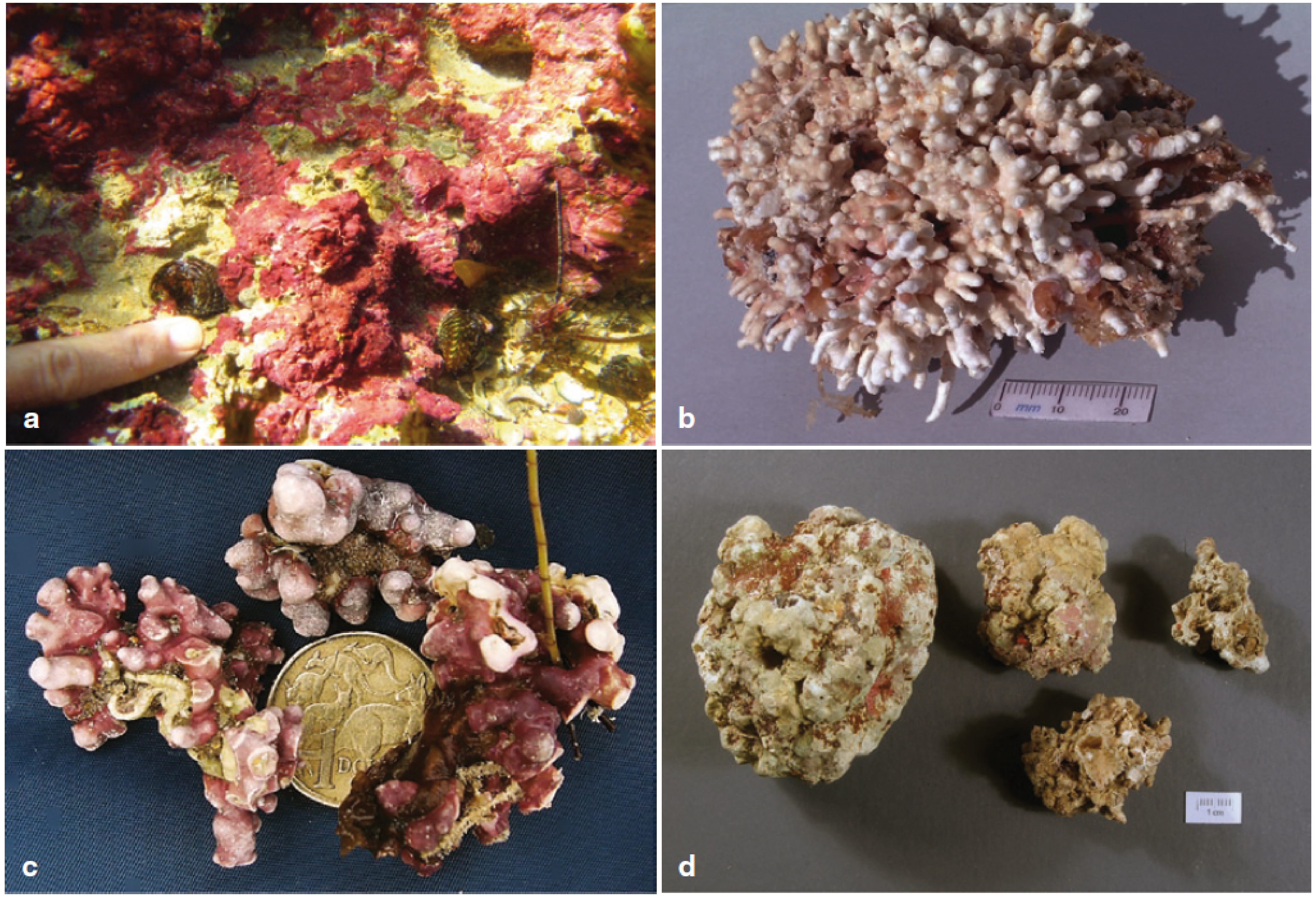


Fig. 4.9 Non-geniculate calcareous red algae. (**a**) Encrusting Pleistocene aeolianite, Robe, S.A. Depth 1 m, finger for scale, (**b**) fruticose rhodolith, Spencer Gulf, 2 mwd, (**c**) fruticose rhodolith, attached to *Amphibolis* stem (coin scale 2 cm in diameter, Spencer Gulf), (**d**) rhodoliths with a smooth nodular growth form; cm scale, Great Australian Bight, Baxter Sector, 46 mwd