EOS 240: Lab Assignment 6

The long term carbon cycle

Due: 2:30 pm March 13, 2025 (Th section)
Due: 1:30 pm March 14, 2025 (F section)

You have one week to complete this assignment. You should submit your response to the course Brightspace page as a single PDF file. **Additionally, we ask that you upload a copy of the scripts, code, or spreadsheets you used to complete the assignment. These documents will help us track down mistakes.** Responses to questions should be typed, using complete sentences and standard grammar. If you choose to support your answers with hand-drawn illustrations or hand-written calculations, you should scan or photograph the written work and integrate it into your PDF file as a figure. Double check that your image resolution is high enough to read. A google search of 'PDF combiner' will return a number of webpages that allow you to upload individual images and combine them into a single .pdf file (example: combinepdf.com). There are also a number of good apps for mobile phones. If you write your response in a word processor, please export to .PDF before submitting your response.

You are not excluded from working with others (pairs are recommended), but each person will submit their own copy of the assignment. In your submission, include the names of anyone you worked with on the assignment.

To answer the questions, you can perform calculations and make figures using Excel (an open source alternative: www.libreoffice.com), or with a program or programming language of choice.

Name : ______________________________

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Marks:	6	4	5	6	21
Score:					

Introduction

In this lab you will build a model of the long term carbon cycle. This model takes the form of a differential equation. You will solve this equation numerically using a truncation of the Taylor series and some algebra. This solution will allow you to simulate different conditions and consider how the model responds.

Question 1 (21)

The long term carbon cycle

Carbon in the ocean and atmosphere plays a major role in regulating tempature of Earth's surface through the greenhouse gas effect as CO_2. The concentration of CO_2 in the atmosphere is proportional to the total mass of carbon in the ocean and atmosphere.

(a) (2 points) Describe the sources and sinks of carbon to and from the ocean and atmosphere on long timescales ($>$ 1 million years). Illustrate your answer as a single box that represents the combined ocean and atmosphere carbon reservoir with mass fluxes coming in and out. Annotate the figure with the chemical reactions that represent the removal of CO_2 from the ocean and atmosphere.

(b) (2 points) If the mass flux of carbon entering the combined ocean and atmosphere box is equal to the flux of carbon leaving the box, the mass of carbon in the box is said to be in *steady state*. If the fluxes are out of balance, then the mass of carbon in the system will change. Describe (conceptually) how the rate of that change may be related to the two mass fluxes and the total mass in the system.

In this next question you will investigate how sensitive the surface carbon reservoir is to minor imbalances in the sources and sinks. This sensitivity, combined with geologic evidence of liquid water on Earth's surface for at least 4 billion years, provides some of the strongest evidence that the long term carbon cycle is regulated by a stabilizing, or negative, feedback.

(c) (2 points) The surface carbon reservoir today contains 70,000 $\times 10^{12}$ mol C, and the volcanic degassing rate is approximately 6,000 $\times 10^{12}$ mol C/ky. If the carbon sink mass flux is 1% more than the carbon source mass flux, how many years does it take to deplete the surface reservoir of carbon? *Hint:*

$$\frac{\partial M}{\partial t} = F_{in} - F_{out}$$

(d) (2 points) A common and simple way to implement a negative feedback in a this sort of chemical box model is to set the outgoing mass flux to be proportional to the total mass in the box:

$$F_{out} \propto M$$
$$F_{out} = k_w M$$

For the long term carbon cycle, we refer to this proportionality coefficient k_w as the *weatherability* of Earth's surface, where a larger k_w represents a more efficient carbon burial system. Assuming that the current Earth surface is in steady state (on million-year timescales), use the reservoir size and volcanic degassing rate from part (c) to determine the weatherability of the present day Earth surface.

(e) (2 points) Describe at least two reasons that weatherability could have changed in the past.

You can determine how the surface carbon reservoir responds to *change* with the following differential equation:

$$\frac{\partial M}{\partial t} = F_{in} - k_w M(t)$$

While calculus provides the tools to determine an analytical solution to this differential equation, it is not a very straight-forward problem. In scientific computing, we often rely on numerical solutions to complex differential equations. Almost all numerical solutions involve a little algebra and Taylor's theorem:

$$f(x + \Delta x) = f(x) + \frac{f'(x)}{1!}(\Delta x) + \frac{f''(x)}{2!}(\Delta x)^2 + ...$$

This theorem says that we can determine an unknown value of a function, $f(x+\Delta x)$, near a known value of that same function, $f(x)$, if we also know the derivatives, $f'(x)$, $f''(x)$, etc., of the function at the known point.

This theorem uses an infinite series of increasingly higher order derivatives of a function $f(x)$. Truncations of this infinite series are referred to as the **k-th order Taylor polynomial** where **k** is equal to the highest order derivative included in the truncated series. These truncations are approximate solutions to the infinite series in the equation above, where higher order polynomials are more accurate across larger discrete steps, Δx. However, if we only consider very small discrete steps, low order Taylor polynomials can be very accurate and allow us numerically solve differential equations.

(f) (2 points) Use the following **1-st order Taylor polynomial** to restate the differential equation above in terms carbon mass at the present, $M(t)$, and carbon mass one discrete step forward in time $M(t + \Delta t)$. Arrange your solution to solve for $M(t + \Delta t)$.
1-st order Taylor polynomial:

$$f(x + \Delta x) \approx f(x) + \frac{f'(x)}{1!}(\Delta x)$$

Hint: the 1-st order Taylor polynomial above is equivalent to:

$$M(t + \Delta t) \approx M(t) + \frac{\partial M}{\partial t}(\Delta t)$$

Set up your numerical solution to the one-box carbon model in excel (or with a programming tool of your choice). For your model, use the volcanic degassing flux from part (c), the weatherability from part (d). Use a discrete timestep (Δt) of 5 ky (or lower) to ensure the stability of your numerical approximation. Start your model with 50,000 Tmols of carbon.

(g) (3 points) Provide a figure showing the model evolution to steady state. Your figure should show time on the x-axis and mols of carbon on the y-axis.

Create two new models that start with the steady-state solution from the previous question. In each new model a catastrophe occurs at 50 ky. In model 1, a planet-changing volcanic event releases 100,000 Tmol of carbon into the ocean and atmosphere. In model 2, a massive impactor collides with Earth and burns most of the terrestrial biosphere, adding 50,000 Tmol of carbon into the ocean and atmosphere.

(h) (3 points) Create a figure that shows how the mass of surface carbon evolves in both models. Your figure should show time on the x-axis, mols of carbon on the y-axis and both models should reach steady-state after the catastrophe.

(i) (2 points) After the 50 ky catastrophes, how much time does it takes each model in part (h) to reach steady state? Does this *relaxation timescale* depend on the size of the mass perturbation?

(j) (1 point) The negative feedback we introduced to our model equation returns the mass of carbon on Earth's surface to steady-state. However, over geologic time-scales we have observed large shifts in Earth's climate. Describe two ways to change the amount of carbon on Earth's surface at steady-state.