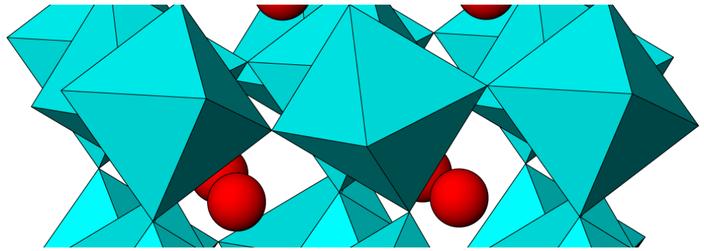
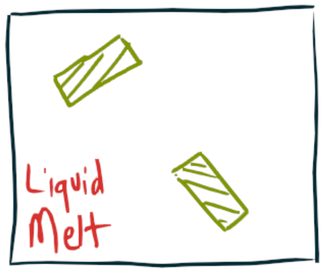


Lecture 7: Batch and fractional crystallization

1. Batch Melting/Crystallization
2. Fractional Crystallization
3. Examples
    A. Olivine crystallizing in a melt
    B. Muskox layered intrusion

We acknowledge and respect the lək̓ʷəŋən peoples on whose traditional territory the university stands and the Songhees, Esquimalt and W̱SÁNEĆ peoples whose historical relationships with the land continue to this day.

Batch melting or crystallization: derivation



closed system : no mass exchange

Subscripts
0 = system
S = solid
L = Liquid Melt

10 kg system with 1 g Ni (extensive)

$\frac{1\,g\,Ni}{10000\,g} = \frac{0.0001\,g\,Ni}{1\,g} = 0.01\,\%\,Ni\ (\times 100) = 100\,ppm\ (\times 10^6)$ (intensive) C = Concentration of trace element

$100\,ppm\,Ni = \frac{0.0001\,g\,Ni}{1\,g} \cdot 10000\,g = 1\,g\,Ni$

$C^{\circ}_{Ni} \cdot M_0 = M^{\circ}_{Ni}$

mass balance

$M_0 = M_S + M_L$

$C_0 M_0 = C_S M_S + C_L M_L$

$\frac{C_0 M_0}{M_0} = \frac{C_S M_S}{M_0} + \frac{C_L M_L}{M_0}$

$\frac{M_L}{M_0} = F$ the fraction of melt

$\frac{M_S}{M_0} = 1 - F$ the fraction of solid

$C_0 = C_S(1-F) + C_L F$

recall $D = \frac{C_S}{C_L}$

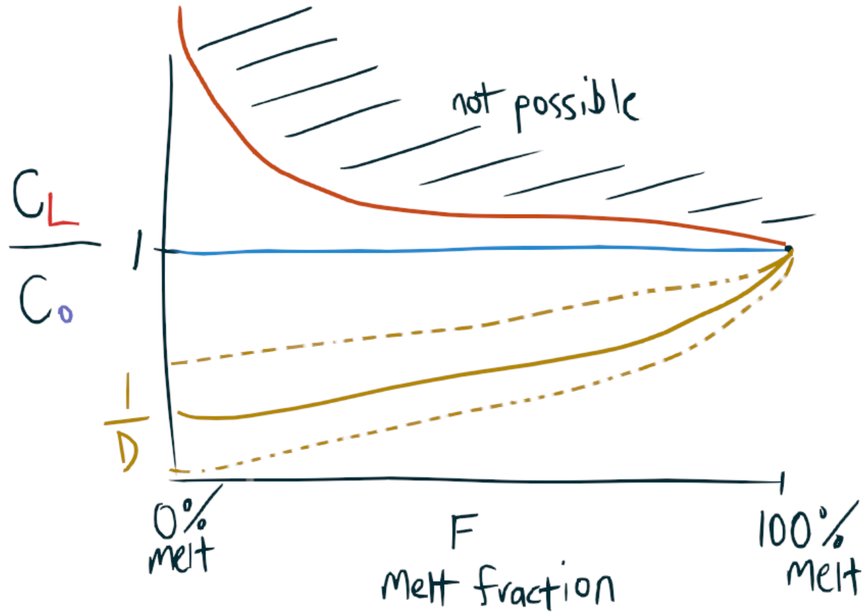
$C_0 = D C_L (1-F) + C_L F$

$C_0 = C_L(F + D(1-F))$

$\frac{C_L}{C_0} = \frac{1}{F + D(1-F)}$ and $\frac{C_S}{C_0} = \frac{D}{F + D(1-F)}$

Batch melting or crystallization: predictions

$$\frac{C_L}{C_0} = \frac{1}{F + D(1-F)}$$

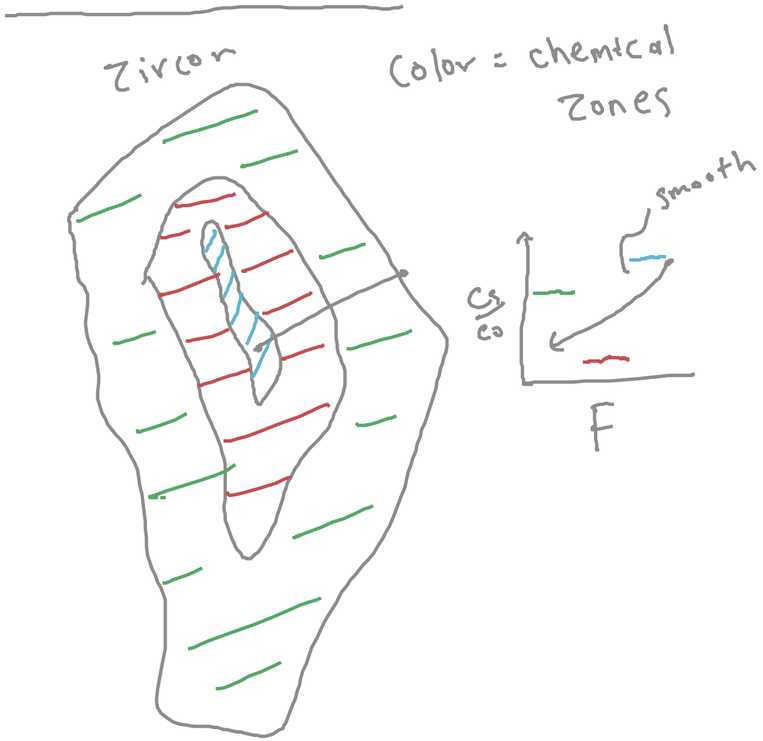


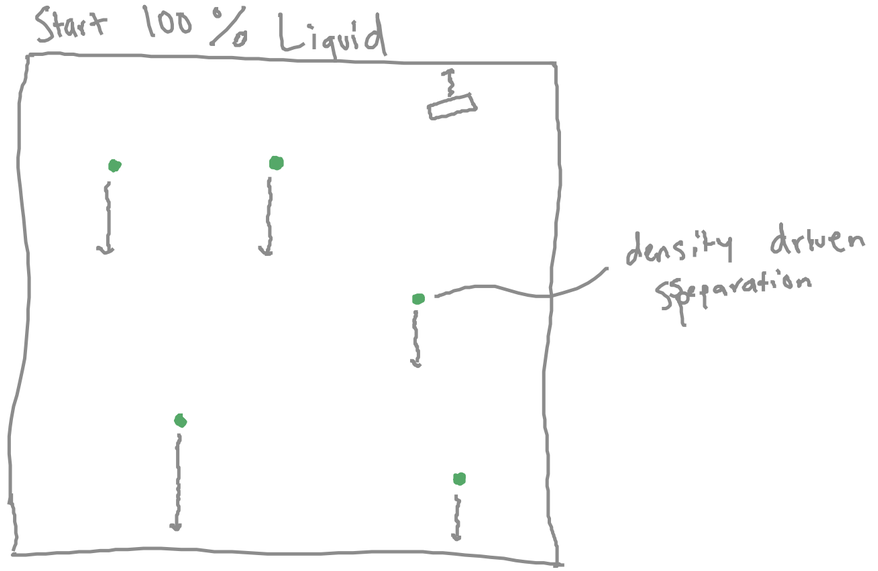
$D = 1$

$D > 1$

$D \simeq 0$

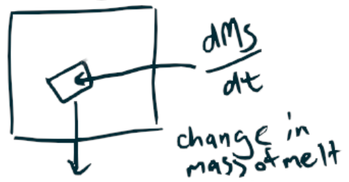
Fractional crystallization: conceptual





Fractional crystallization: derivation

Starting with 100% melt



open system

rate of crystal segregation

$$\frac{dM_L}{dt} \neq 0 = -\frac{dM_S}{dt} \quad (1)$$

product rule

infinitely small crystal in equilibrium with melt

$$d(C_L M_L) = dC_L M_L + dM_L C_L = -C_S \cdot dM_S$$

using $C_S = C_L \cdot D$ and (1)

$$dC_L M_L + dM_L C_L = C_L \cdot D\, dM_L$$

$$dC_L M_L = C_L \cdot D\, dM_L - dM_L C_L$$

$$dC_L M_L = (D-1)\, C_L\, dM_L$$

$$\int_{C_0}^{C_L} \frac{dC_L}{C_L} = (D-1) \int_{M_0}^{M_L} \frac{dM_L}{M_L}$$

$$\ln \frac{C_L}{C_0} = (D-1) \ln \frac{M_L}{M_0}$$

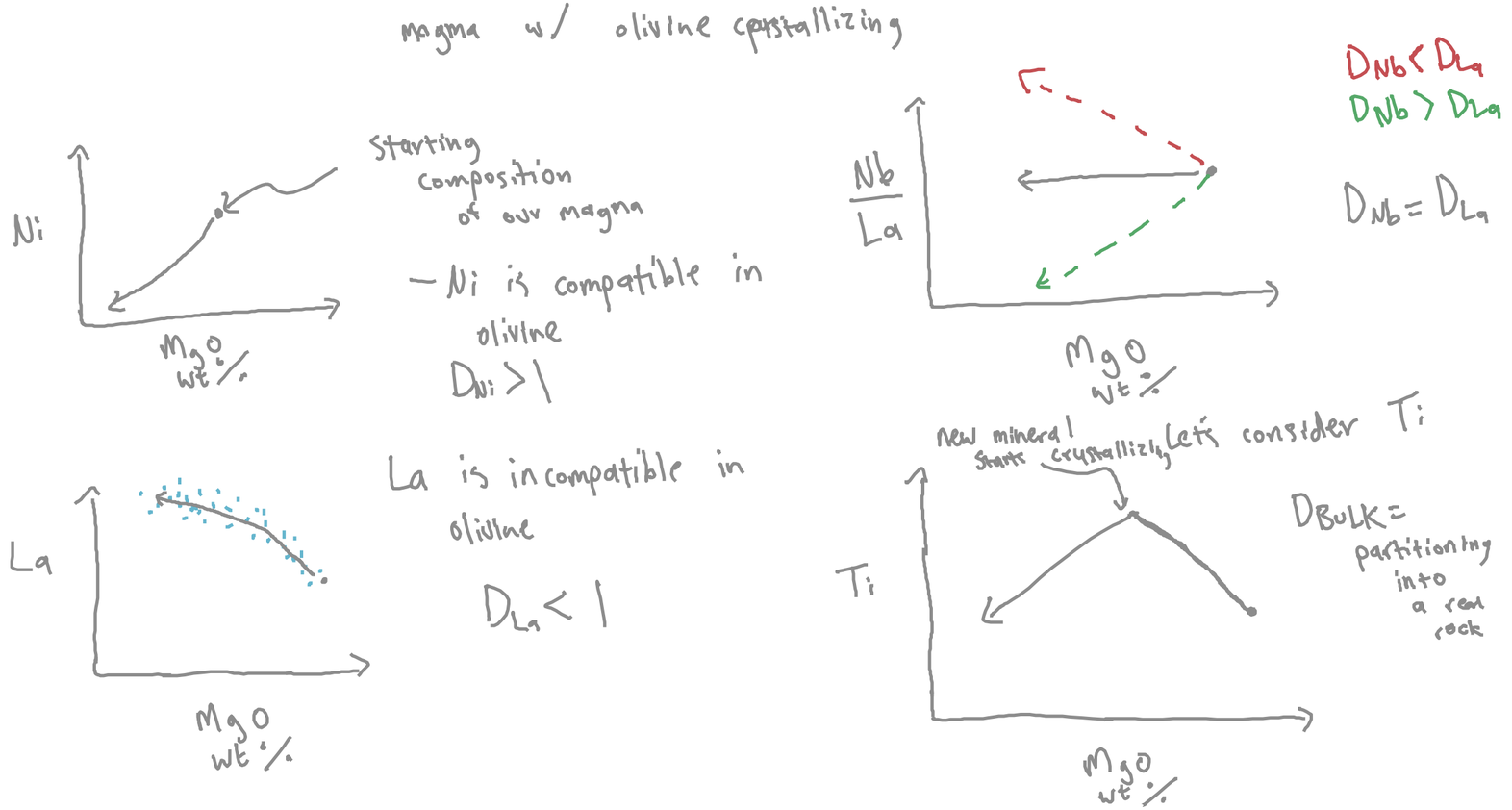
$$\ln \frac{C_L}{C_0} = (D-1) \ln F$$

$$\ln \frac{C_L}{C_0} = \ln F^{D-1}$$

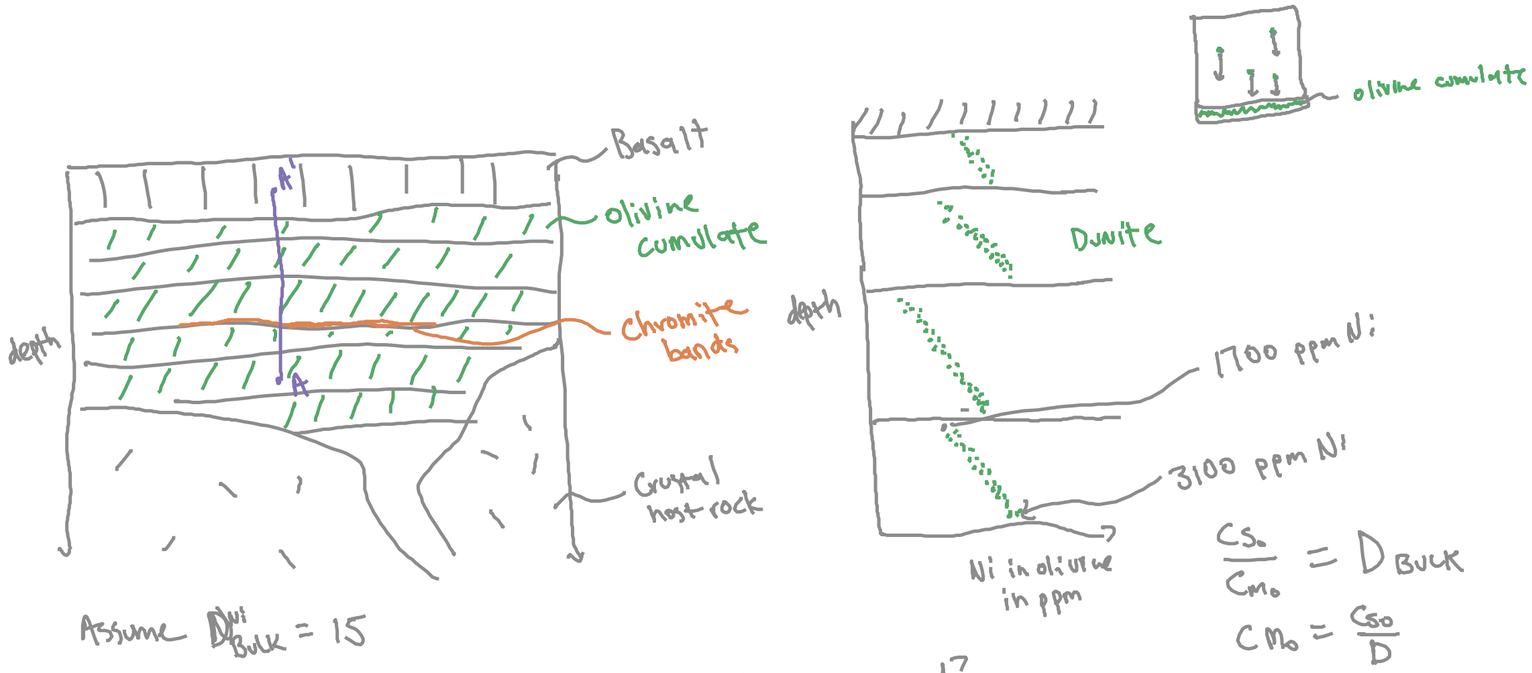
$$\frac{C_L}{C_0} = F^{D-1}$$

$$\frac{C_S}{C_0} = D \cdot F^{D-1}$$

Fractional crystallization: examples



Muskox Layered Intrusion Example



Assume $D^{Ni}_{BULK} = 15$

$\frac{C_{S_0}}{C_{M_0}} = D_{BULK}$

$C_{M_0} = \frac{C_{S_0}}{D}$

From 3100 ppm to 1700 ppm, how much melt crystallized?

$\frac{C_M}{C_{M_0}} = F^{D-1}$ $\frac{C_S}{C_{M_0}} = D \cdot F^{D-1}$ $\frac{1700}{\left(\frac{3100}{15}\right)} = 15 \cdot F^{15-1}$