Lecture 14: The Long Term Carbon Cycle

1. The temperature of the Earth
2. A faint young Sun
3. The necessity of a negative feedback
4. The (long-term) Carbon cycle
5. Finite-difference solutions to differential equations

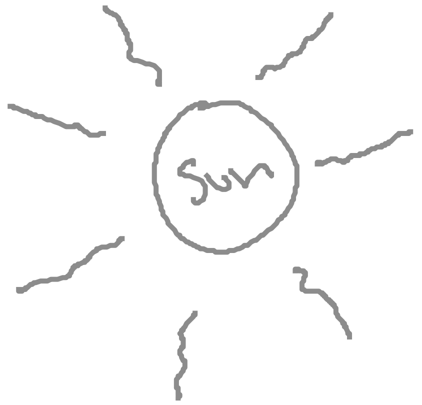
We acknowledge and respect the lək̓ʷəŋən peoples on whose traditional territory the university stands and the Songhees, Esquimalt and W̱SÁNEĆ peoples whose historical relationships with the land continue to this day.

What sets the temperature of the Earth?

of any planet?

- incoming radiation
- interior heating (radioactive decay)

3.87×10^{26} W $= Q$



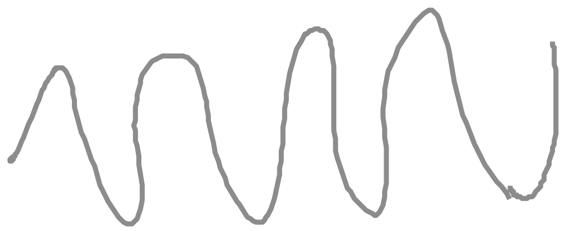
how much energy does Earth see?

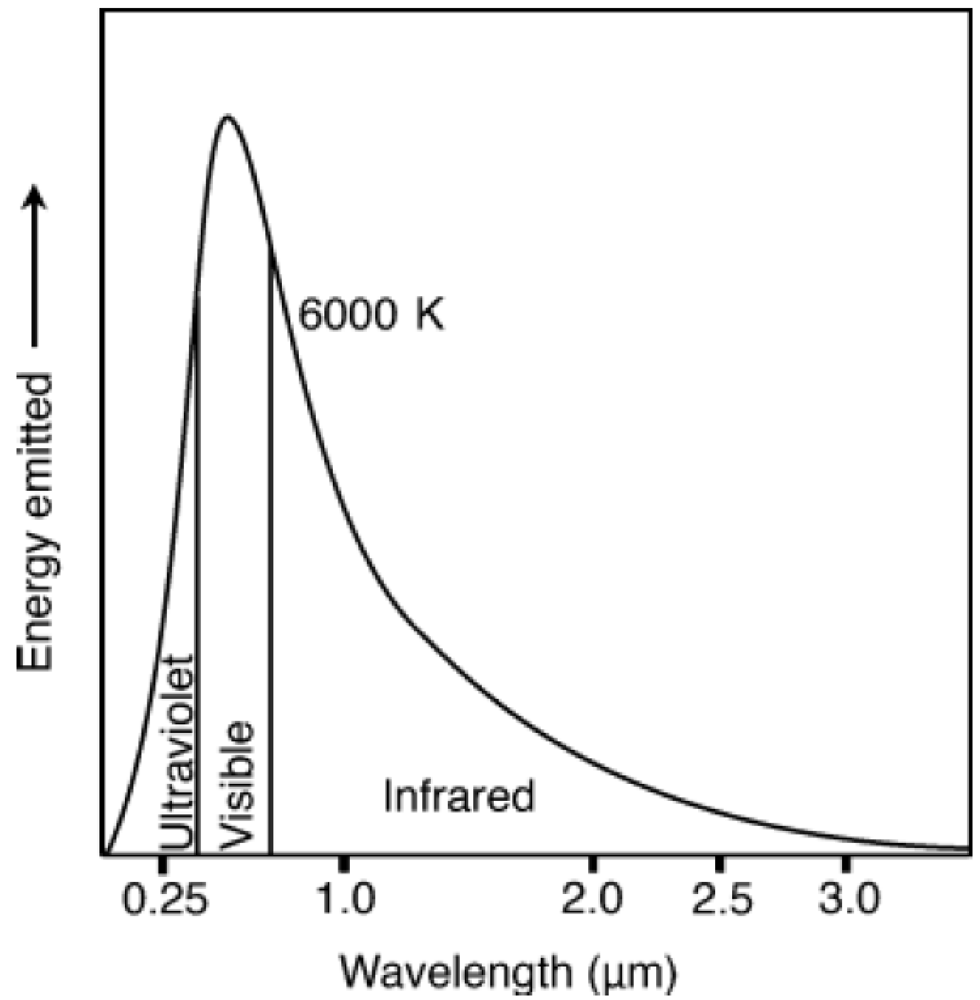
– function of radius

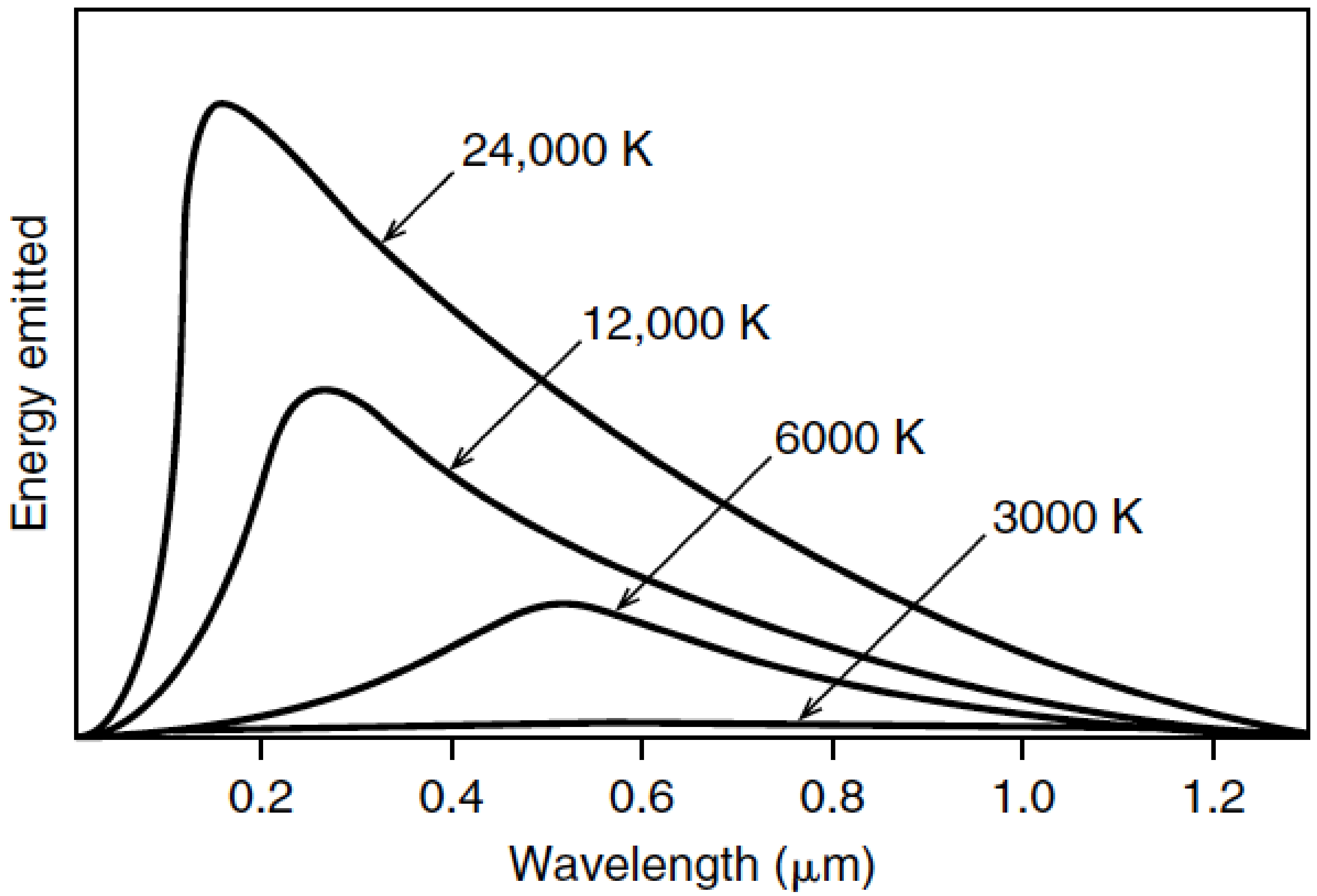
$\frac{Q}{4\pi r^2} = S_0$ ← solar constant

$S_0 = 1367 \ W \ m^{-2}$ for Earth

Spectrum of frequencies



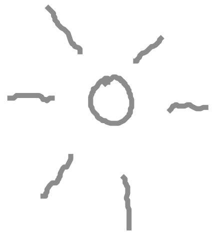

Energy emitted
6000 K
Ultraviolet
Visible
Infrared
0.25
1.0
2.0
2.5
3.0
Wavelength (μm)


Energy emitted
24,000 K
12,000 K
6000 K
3000 K
0.2
0.4
0.6
0.8
1.0
1.2
Wavelength (μm)

Calculating the 'emission' temperature of Earth

- incoming radiation
- interior heat (radio active decay)
- other heat from accretion

$Q = 3.87 \times 10^{26}$ W

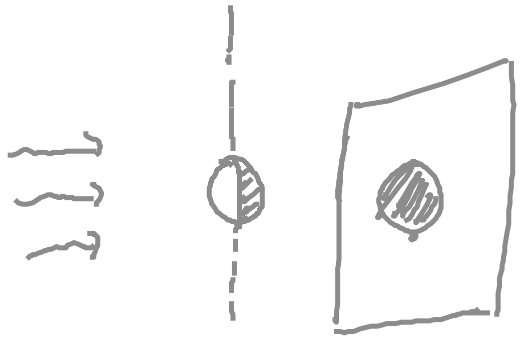


Solar constant $= S_0$

$\dfrac{Q}{4\pi r^2} = S_0$

distance from sun

$S_0 = 1367 \text{ W m}^{-2}$ for Earth today



πr^2

Earth radius

incoming radiation on the surface

$S_0 \cdot \pi r^2 (1-\alpha)$

α = albedo

outgoing radiation

σT_E^4

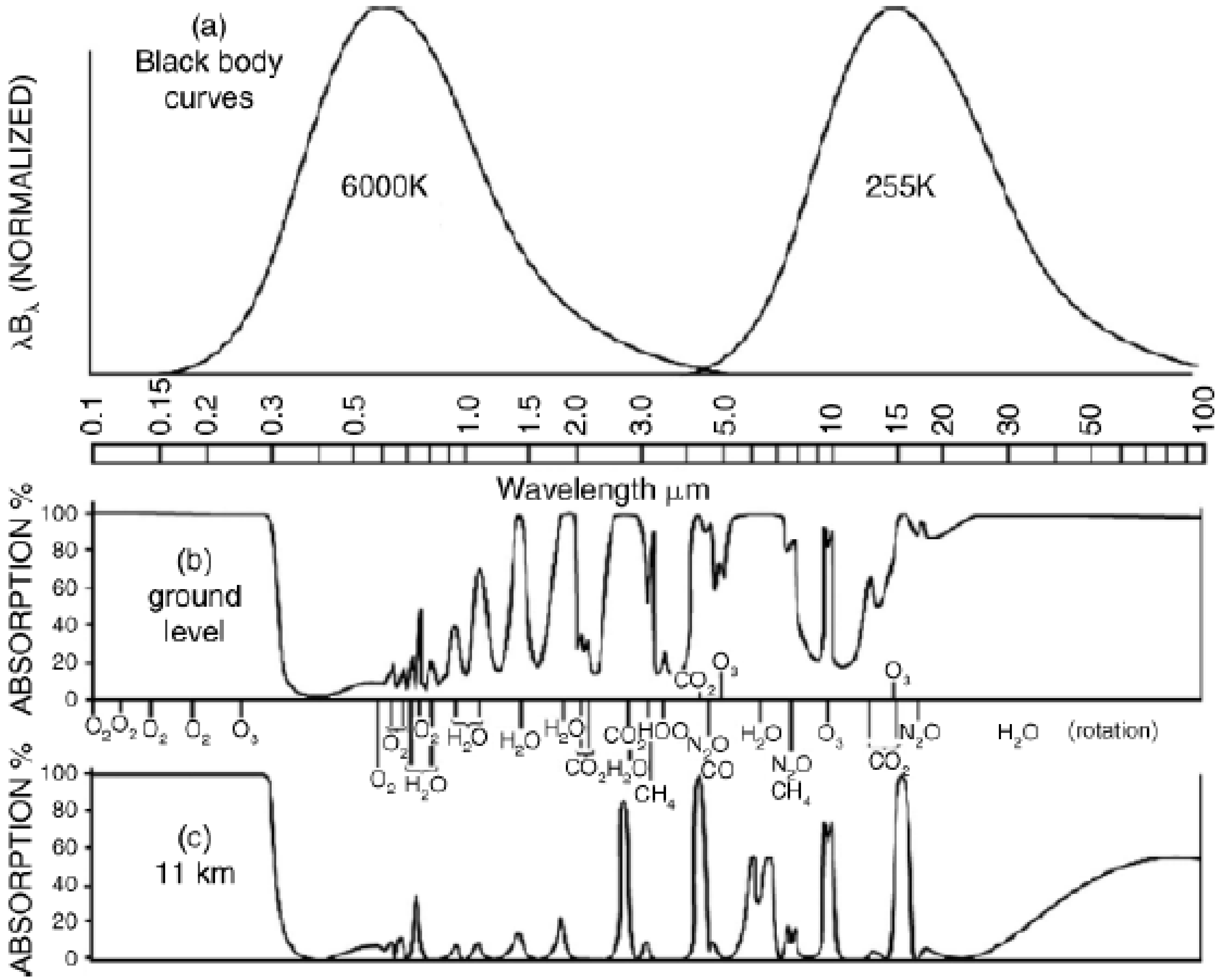
Stefan-Boltzman law

$\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

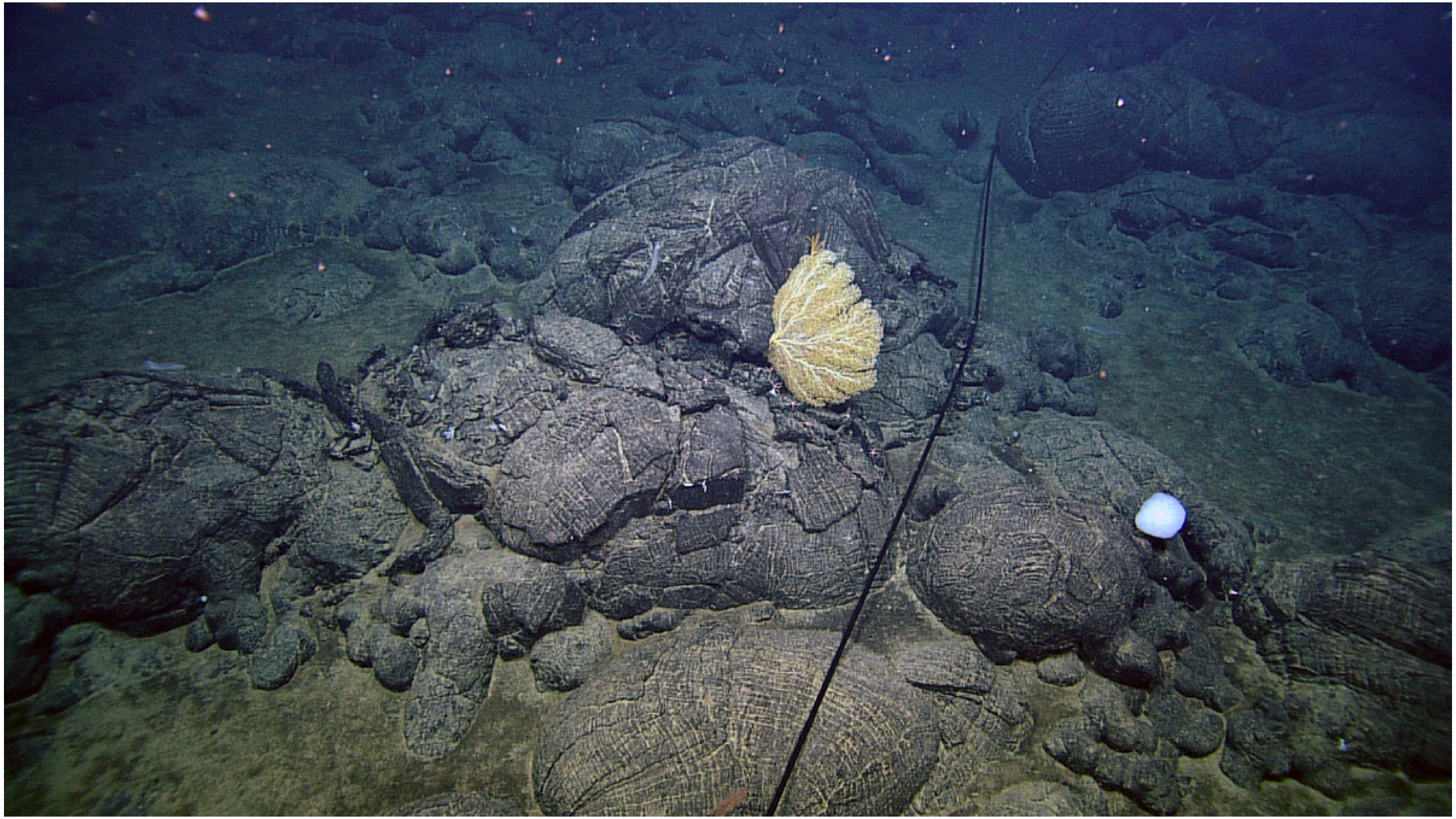
$T_E = 255$ K ($-18°$c)

$T_s = 288$ K ($15°$c)

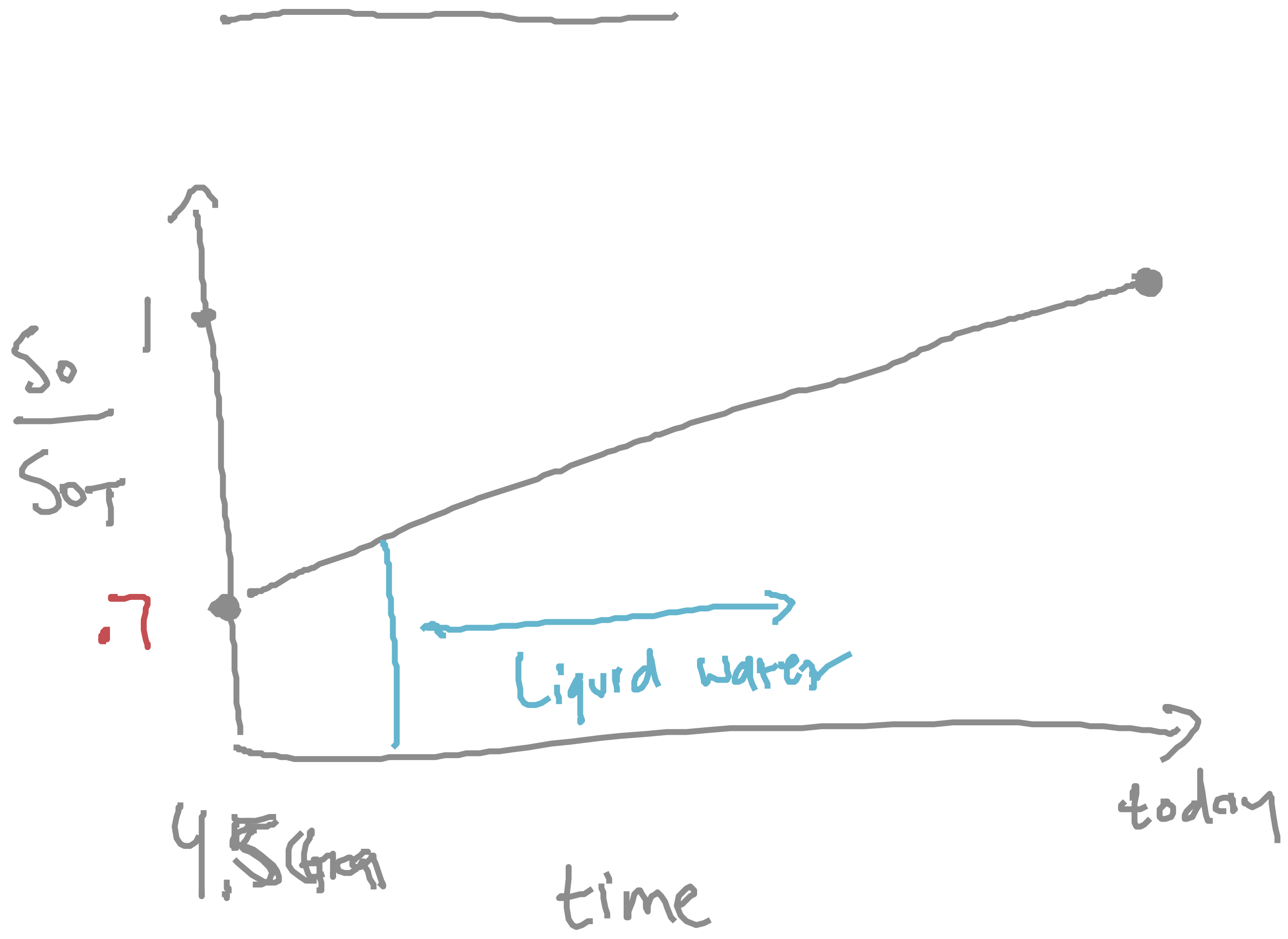
- Difference is the "greenhouse" effect



(a)
Black body curves
6000K
255K
λB_λ (NORMALIZED)
0.1 0.15 0.2 0.3 0.5 1.0 1.5 2.0 3.0 5.0 10 15 20 30 50 100
Wavelength μm
(b)
ground level
ABSORPTION %
100 80 60 40 20 0
(c)
11 km
ABSORPTION %
H2O (rotation)



A faint young Sun and possible solutions



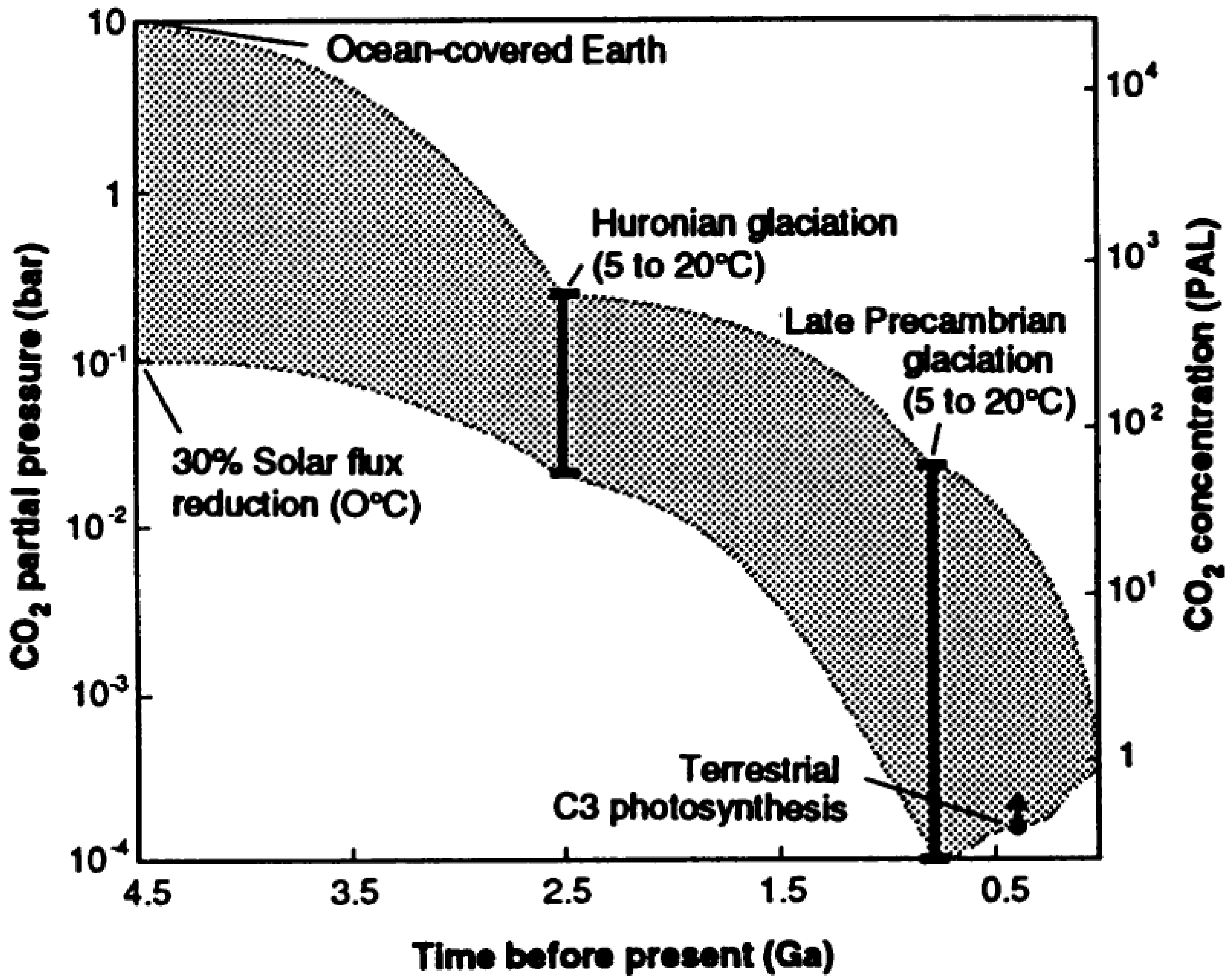
Solar constant increased by 30 %

↳ we need 72 W/m²

- Larger GHG effect
- radioactive decay

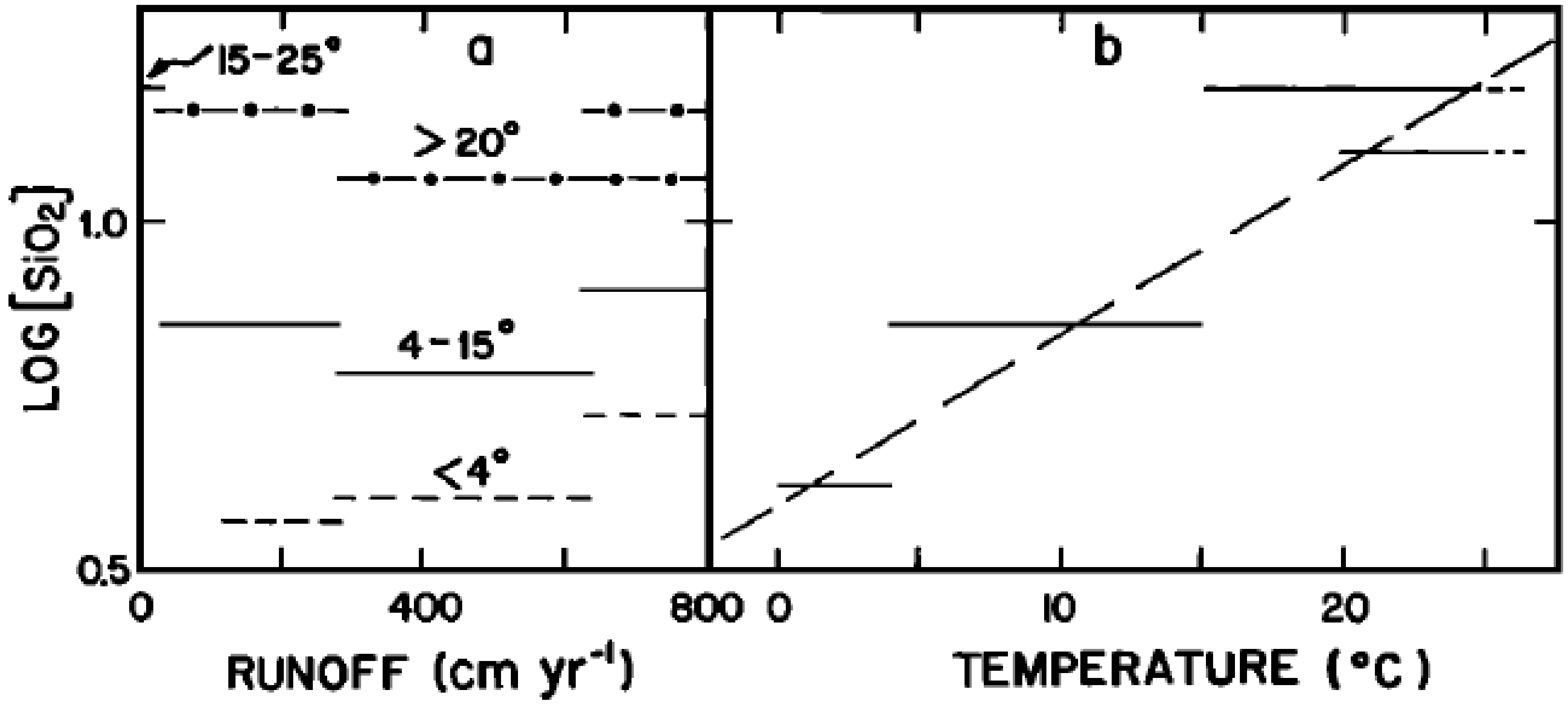
  today: 0.06 W/m²

  4.5 Ga: 0.3 W/m²

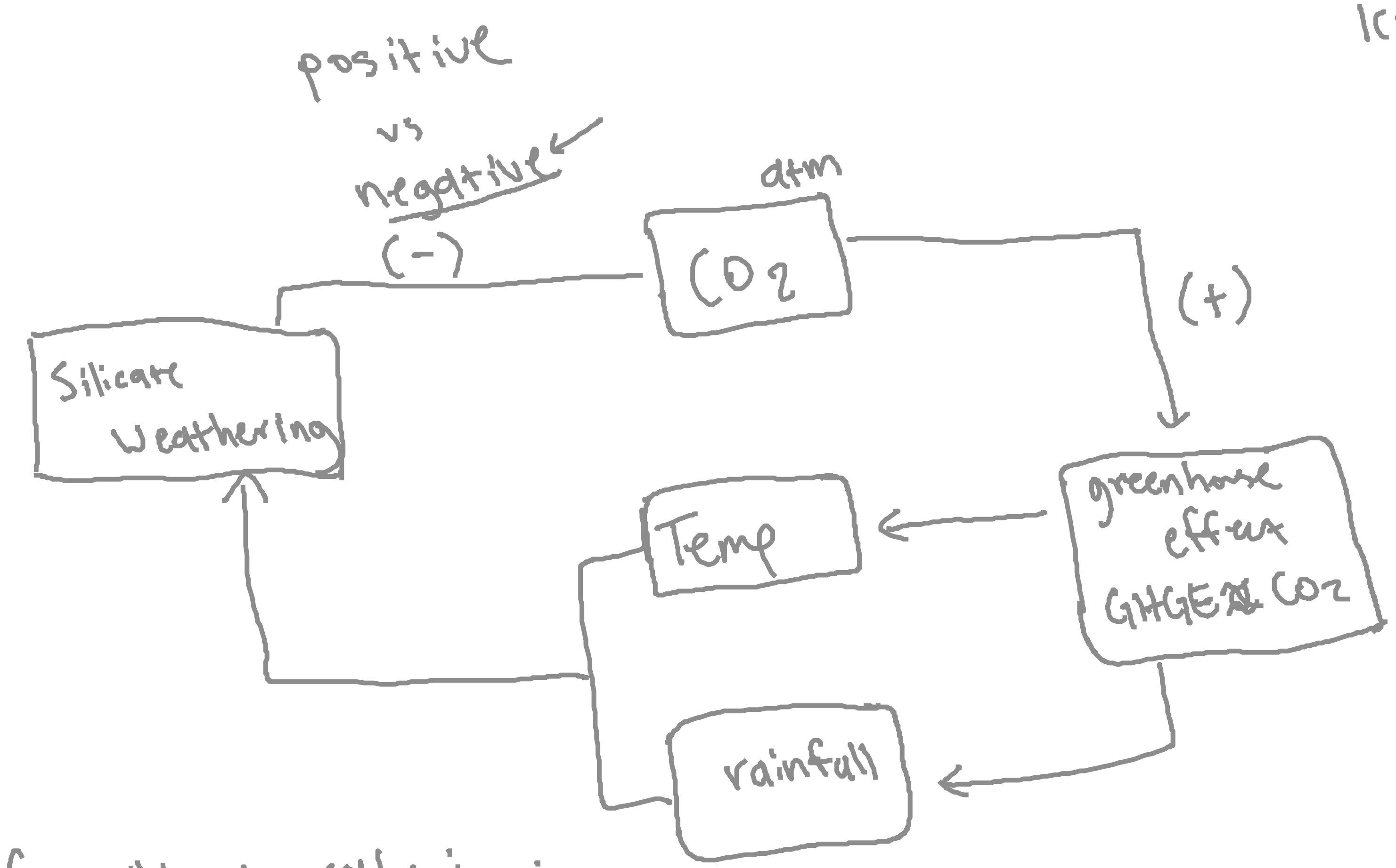


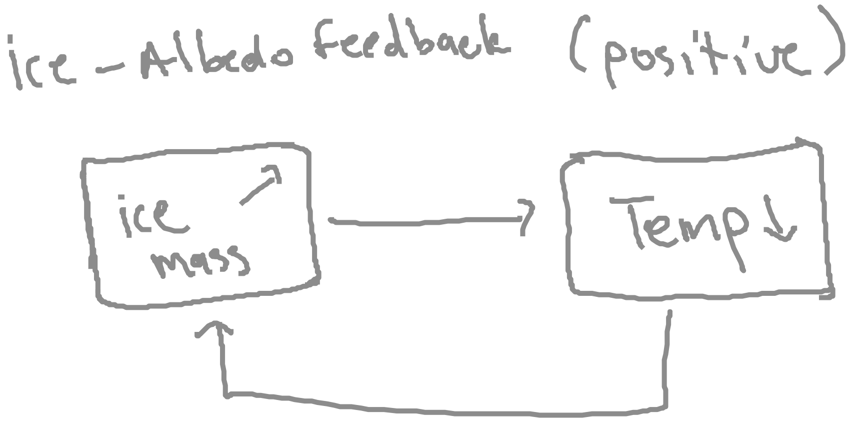
Ocean-covered Earth
Huronian glaciation
(5 to 20°C)
Late Precambrian
glaciation
(5 to 20°C)
30% Solar flux
reduction (O°C)
Terrestrial
C3 photosynthesis
10
1
10-1
10-2
10-3
10-4
10^4
10^3
10^2
10^1
1
4.5
3.5
2.5
1.5
0.5
CO2 partial pressure (bar)
CO2 concentration (PAL)
Time before present (Ga)

The silicate weathering feedback



Feedbacks





if silicate weathering:

↗ then CO_2 ↘

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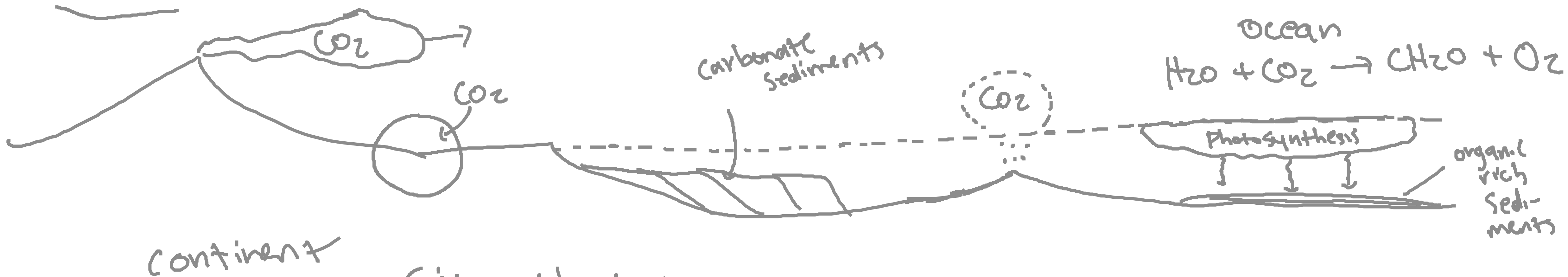
The long term carbon cycle (conceptual)

million – billion years

$$CO_2 + H_2O \longleftrightarrow H_2CO_3$$



Silicate Weathering

$$CaSiO_3 + 2\,H_2CO_3 \longrightarrow \underline{Ca^{2+} + 2HCO_3^-} + SiO_2 + H_2O$$

Carb. weathering

└→ carb. precipitation

no change in CO_2

Carbonate Weathering

$$CaCO_3 + H_2CO_3 \underset{\text{precipitation}}{\overset{\text{carbonate weathering}}{\rightleftharpoons}} \underline{Ca^{2+} + 2HCO_3^-}$$

Silicate weathering

└ $-1CO_2$

└→ carb. precipitation

The long term carbon cycle (a model)

box model

Steady state: mass doesn't change over time

Volcanic degassing → Carbon mass ocean + atmosphere → Carbonates and organic sediments

K: weatherability "efficiency of weathering"

Flux out $= K \cdot M$

$$\frac{dM}{dt} = \text{Flux in} - \text{Flux out}$$

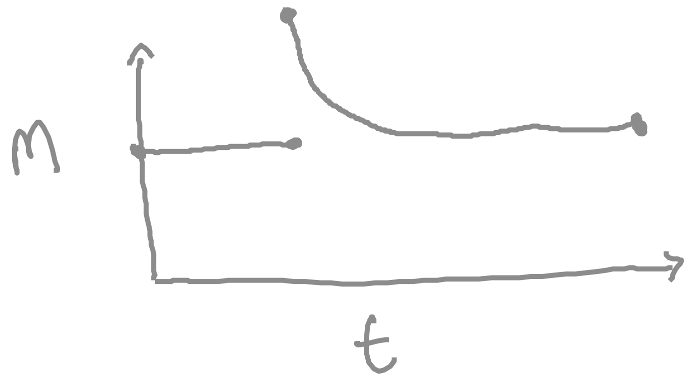
$\frac{dM}{dt} = 0$ when we have steady state

w/ a negative feedback

$$\frac{dM}{dt} = \text{Flux in} - K \cdot M$$

$$0 = \text{Flux in} - K \cdot M$$

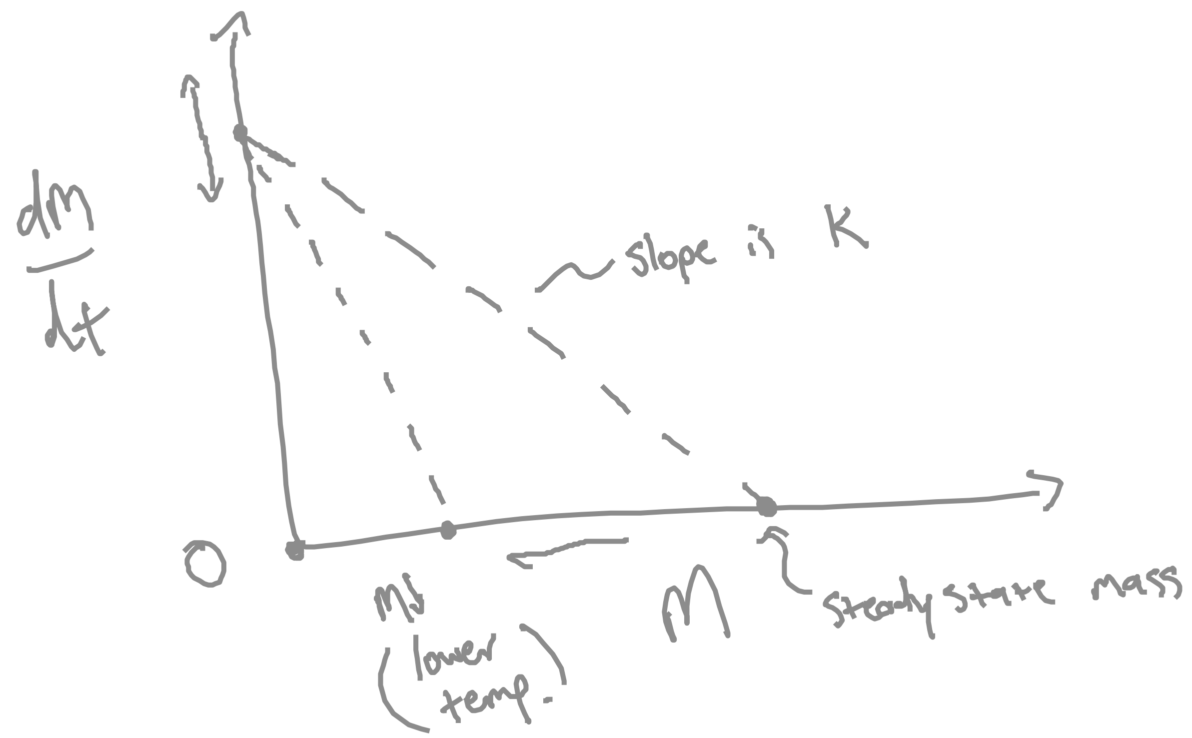
$$K M = \text{flux in}$$



How does climate change?

- change steady state M

$$\frac{dM}{dt} = F_{in} - KM \qquad (Y = b - mx)$$



The Derivative Function

Recall: what is the definition of the derivative function $f'(x)$?

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{\Delta f}{\Delta x}$$

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While we can't calculate Δx and Δf for $\lim\,(\Delta x \to 0)$ CPUs have no trouble calculating $\frac{\Delta f}{\Delta x}$ for a sufficiently small value of Δx

Finite difference methods

In fact, we have many possible approaches to estimating derivatives using *sufficiently small* values of Δx, and these methods are collectively known as **finite difference methods**. These methods make use of Taylor's theorem:

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What happens to the size of each higher order term in the series?

We describe the error of an approximation by the degree of the term where the series is truncated. First order: $O(\Delta x)$, second order: $O(\Delta x^2)$, third order: $O(\Delta x^3)$, etc...

$$f(x + \Delta x) \approx f(x) + \frac{f'(x)}{1!}(\Delta x) + \frac{f''(x)}{2!}(\Delta x)^2 + \ldots$$

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We can use small Δx and a first order truncation of the Taylor series to estimate $f(x+\Delta x)$